AP Calculus BC: The Awesome Sheet of Knowledge

Parametrics

Position: $\langle x(t), y(t)\rangle$

Position: $x(b) = x(a) + \int_a^b x'(t)dt$

Velocity: $\langle x'(t), y'(t)\rangle$ or $\langle \frac{dx}{dt}, \frac{dy}{dt}\rangle$

Acceleration: $\langle x''(t), y''(t)\rangle$

Speed: $\sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}$

Slope of Tangent Line: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

Polar

Area $= \frac{1}{2}\int_a^b r^2 d\theta$

Slope of Tangent Line: $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$x = r\cos(\theta)$

$y = r\sin(\theta)$

Error Bounds

Alternating: Error $< |a_{n+1}|$

Lagrange: Look at next term of Taylor series and estimate a max value for derivative.

$$\text{Error} \leq \left|\frac{\max\left(f^{n+1}(z)\right) x^{n+1}}{(n+1)!}\right|$$

Theorems

IVT: guarantees a point.

1. State the end points (point should be between endpoints)
2. Explanation: Since (**given function**) is continuous on (**given interval**), IVT guarantees that (**repeat wordi**

MVT: guarantees a derivative value

1. Find slope between the endpoints.
2. Explanation: Since (**given function**)is continuous AND differentiable on (**given interval**), MVT guarantees that (**repeat wording in the question**)

Note: Differentiability implies continuity!

Know these Power Series

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots \qquad \text{If} -1 < x < 1$$

Taylor Polynomials

The nth degree Taylor Polynomial to approximate $f(x)$:

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)(x-c)^2}{2!} + \cdots + \frac{f^{(n)}(c)(x-c)^n}{n!}$$

Find $f'(x)$: Make sure polynomial or series is expanded and differentiate each term individually.

Find $\int_a^x f(t)dt$: Make sure polynomial or series is expanded and anti-differentiate each term individually.

Remember: "x" is the variable!

Coefficient of nth term $= \frac{f^{(n)}(c)}{n!}$

Euler's Method

$$y_{n+1} = y_n + h * f'(x_n, y_n)$$

Where h is the step size (can be negative!)

Logistic Growth

$$\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$$

L is the carrying capacity.

Fastest growth occurs when $y = \frac{L}{2}$

Graph is CU when $y < \frac{L}{2}$ and CD when $y > \frac{L}{2}$

Arc Length

Rectangular (x, y): Arc Length $= \int_a^b \sqrt{1 + [f'(x)]^2}dx$

Note: could be used to find perimeter in an Area/Volume problem

Parametrics: Arc Length $= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Area/Volume (AB)

Area: $\int_a^b$ (Area of representative rectangle)

Volume by washers: $V = \pi \int_a^b R^2 - r^2$

Note: Look for the phrase "rotated around the line … "

Volume by cross sections: $V = \int_a^b$ (Area of cross section)

Note: Look for the phrase "cross sections perpendicular … "

General Notes: Always draw representative rectangle first!!!
Top – Bottom or Right – Left
For Volume: Rectangles **MUST** be perpendicular to axis/line

Derivative Rules

$f(u) = \sin(u)$
$f'(u) = \cos(u) * u'$

$f(u) = \tan(u)$
$f'(u) = \sec^2(u) * \mathrm{u}'$

$f(u) = \sec\,(u)$
$f'(u) = \sec(u)\tan(u) * u'$

$f(u) = \cos(u)$
$f'(u) = -\sin(u) * u'$

$f(u) = \cot(u)$
$f'(u) = -\csc^2(u) * u'$

$f(u) = \csc\,(u)$
$f'(u) = -\csc(u)\cot(u) * u'$

$f(u) = \ln(u)$
$f'(u) = \frac{u'}{u}$

$f(u) = e^u$
$f'(u) = e^u * u'$

$f(u) = a^u$
$f'(u) = a^u * \ln(a) * u'$

$f(u) = \tan^{-1}(u)$
$f'(u) = \frac{u'}{1+u^2}$

$f(u) = \sin^{-1}(u)$
$f'(u) = \frac{u'}{\sqrt{1-u^2}}$

$f(u) = \sec^{-1}(u)$
$f'(u) = \frac{u'}{|u|\sqrt{u^2-1}}$

Integration Rules

$\int \sin(u)\,du = -\cos(u) + C$

$\int \cos(u)\,du = \sin(u) + C$

$\int e^u du = e^u + C$

$\int u^{-1} du = \ln|u| + C$

$\int u^n du = \frac{u^{n+1}}{n+1} + C$

$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

$\int \frac{du}{\sqrt{a^1-u^2}} = \sin^{-1}\frac{u}{a} + C$

$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{|u|}{a} + C$

Partial Fractions: $\int \frac{1}{(x-c)(x-d)}dx = \int \left(\frac{A}{x-c} - \frac{B}{x-d}\right)dx$

Integration by Parts: $\int u \cdot dv = uv - \int v \cdot du$

Convergence Tests

nth term test for DIVERGENCE: Given $\sum a_n$ ... If $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges

ratio test: $\sum a_n$ converges absolutely when $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$ **NOTE**: You must also check endpoints

***p*-series test**: $\sum \frac{1}{n^p}$ converges when $p > 1$

geometric series test: $\sum a(r)^n$ converges when $|r| < 1$

limit comparison test: compare a series to a basic series. Transformation will not affect the convergence of the series.

Alternating Series Test: If a series $\sum(-1)^n a_n$, is alternating, then if $\lim_{n\to\infty} a_n = 0$, then $\sum(-1)^n a_n$ converges.

Absolute vs. Conditional Convergence: Consider with alternating series! Given $\sum(-1)^n a_n$ converges, if:

1. $\sum a_n$ converges then $\sum(-1)^n a_n$ converges **ABSOLUTELY**
2. $\sum a_n$ diverges then $\sum(-1)^n a_n$ converges **CONDITIONALLY**