LIMITS

Basic Limits

The limit of a constant is a constant.
If k is any constant,

$$\lim_{x \to a} k = k \qquad \lim_{x \to +\infty} k = k \qquad \lim_{x \to -\infty} k = k$$

$$\lim_{x \to a} x = a \qquad \lim_{x \to +\infty} x = +\infty \qquad \lim_{x \to -\infty} x = -\infty$$

Properties of Limits

$\lim[f(x) \pm g(x)] = \lim f(x) \pm \lim g(x)$ — The limit of a sum/difference is the sum/difference of the limits.

$\lim[f(x) \cdot g(x)] = \lim f(x) \cdot \lim g(x)$ — The limit of a product is the product of the limits.

$\lim \frac{f(x)}{g(x)} = \frac{\lim f(x)}{\lim g(x)}, \quad \lim g(x) \neq 0$ — The limit of a quotient is the quotient of the limits provided the limit of the denominator is nonzero.

$\lim \sqrt[n]{f(x)} = \sqrt[n]{\lim f(x)}$ — The limit of the nth root is the nth root of the limit.

$\lim[f(x)]^n = [\lim f(x)]^n$ — The limit of a power is the power of the limit.

$\lim k \cdot f(x) = k \cdot \lim f(x)$ — A constant factor can be moved through a limit sign.

Power Functions

$$\lim_{x \to +\infty} x^n = +\infty, \qquad n = 1,2,3,\ldots$$

$$\lim_{x \to -\infty} x^n = \begin{cases} +\infty & n = 2,4,6,\ldots \\ -\infty & n = 1,3,5,\ldots \end{cases}$$

Limits of Polynomials

For any polynomial $p(x) = c_0 + c_1x + c_2x^2 + \ldots + c_nx^n$ and any real number a,

$$\lim_{x \to a} p(x) = \lim_{x \to a}\left(c_0 + c_1x + c_2x^2 + \ldots + c_nx^n\right) = c_0 + c_1a + c_2a^2 + \ldots + c_na^n = p(a)$$

Remember that polynomials behave like its term of highest degree as $x \to +\infty$ *or* $x \to -\infty$.

$$\lim_{x \to +\infty}(c_0 + c_1x + c_2x^2 + \ldots + c_nx^n) = \lim_{x \to +\infty} c_nx^n$$

$$\lim_{x \to -\infty}(c_0 + c_1x + c_2x^2 + \ldots + c_nx^n) = \lim_{x \to -\infty} c_nx^n$$

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Rational Functions

For $\lim_{x \to a} \frac{f(x)}{g(x)}$, there are three cases to consider.

1. If $g(a)$ is nonzero.

   This problem reduces to the limit of a quotient is the quotient of the limits.

   $$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$$

2. If $f(a)$ and $g(a)$ are both equal to zero.

   The numerator and denominator have a common factor of $x - a$.
   Simplify the numerator and denominator and then cancel the common factors.

3. If $f(a)$ is nonzero and $g(a)$ is equal to zero.

   The $\lim_{x \to a} \frac{f(x)}{g(x)}$ does not exist (D.N.E.).
   Analyze the sign of the ratio using a sign chart or sketch and analyze the graph. Look for an answer of $\pm\infty$ or D.N.E..

Rational Functions continued

For $\lim_{x \to +\infty} \frac{f(x)}{g(x)}$ or $\lim_{x \to -\infty} \frac{f(x)}{g(x)}$ consider the terms of highest degree in the numerator and denominator.

$$\lim_{x \to +\infty} \frac{f(x)}{g(x)} = \lim_{x \to +\infty} \frac{c_0 + c_1 x + c_2 x^2 + \ldots + c_n x^n}{d_0 + d_1 x + d_2 x^2 + \ldots + d_m x^m} = \lim_{x \to +\infty} \frac{c_n x^n}{d_m x^m}$$

$$\lim_{x \to -\infty} \frac{f(x)}{g(x)} = \lim_{x \to -\infty} \frac{c_0 + c_1 x + c_2 x^2 + \ldots + c_n x^n}{d_0 + d_1 x + d_2 x^2 + \ldots + d_m x^m} = \lim_{x \to -\infty} \frac{c_n x^n}{d_m x^m}$$