What Do We Need to Know ?

Common Antiderivative Forms for the AP Calculus Exam		
General Power Rule $\int x^n dx = \frac{x^{n+1}}{n+1} + C$		**Natural Log Form** $\int \frac{1}{x} dx = \ln\lvert x\rvert + C$
$\int e^x dx = e^x + C$	**Exponential Functions**	$\int a^x dx = \frac{1}{\ln a} \cdot a^x + C$
$\int \cos x\, dx = \sin x + C$	**Basic Trigonometric Functions**	$\int \sin x\, dx = -\cos x + C$
$\int \sec^2 x\, dx = \tan x + C$		$\int \csc^2 x\, dx = -\cot x + C$
$\int \sec x \cdot \tan x\, dx = \sec x + C$		$\int \csc x \cdot \cot x\, dx = -\csc x + C$
$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$	**Inverse Trigonometric Forms**	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$

Topic Name	Topic #
Applying Properties of Definite Integrals	6.6
Integrating Using Substitution	6.9
Integrating Functions Using Long Division and Completing the Square	6.10
Integration Using **Integration by Parts ***	6.11
Using (Nonrepeating) **Linear Partial Fractions ***	6.12
Evaluating Improper Integrals *	6.13

1. Level: AP 2

$$\int \frac{3x+1}{x^2-x-6}\,dx = \int\left(\frac{A}{x-3}+\frac{B}{x+2}\right)dx$$

Find $A+B$

2. Level: AP 3

$$\int_1^{\infty} \frac{2}{x^3}\,dx =$$

Integration by Parts

If u and v are functions of x and have continuous derivatives, then

$$\int u\,dv = uv - \int v\,du\,.$$

Guidelines for Choosing "*u*" and "*dv*".

The function piece of the integrand you choose for "*u*" should be made according to this mnemonic: L.I.A.T.E.

Logarithm Inverse Trigonometric Algebraic Trigonometric Exponential

3. Level: AP 2

$$\int 2x\sin(x)\,dx$$

Find u and dv

4. Level: AP 3

$$\int \frac{1}{x^2+4x-21}\,dx =$$

(A) $\frac{1}{10}\ln\left|\frac{x-3}{x+7}\right|+C$

(B) $\frac{1}{5}\arctan\left(\frac{x+2}{5}\right)+C$

(C) $\frac{1}{10}\ln\left|\frac{x+7}{x-3}\right|+C$

(D) $\frac{1}{10}\left(\frac{1}{x+7}-\frac{1}{x-3}\right)+C$

5. Level: AP 3

It is known that $\int f(x)\sec^2 x\,dx = f(x)\cdot\tan x - \int 6x^2 \tan x\ dx.$ Which of the following could be $f(x)$?

(A) $18x\cdot\sec^2 x$ (B) $18x$ (C) $2x^3\cdot\sec x\cdot\tan x$ (D) $2x^3$

6. Level: AP 4

$$\int_1^4 \frac{1}{x-3}dx=$$

(A) $-\ln(2)$ (B) $\frac{3}{2}$ (C) $\frac{3}{20}$ (D) The integral diverges.

7. Level: AP 4

$\int x\cos(2x)dx =$

(A) $\frac{1}{4}x^2\sin(2x)+C$

(B) $2x\sin(2x)+4\cos(2x)+C$

(C) $\frac{1}{2}x\sin(2x)-\frac{1}{2}\sin(2x)+C$

(D) $\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)+C$

8. Level: AP 4

$\int_2^5 \frac{6x}{(x+2)(x-1)}dx =$

(A) $-\frac{27}{14}$

(B) $\ln\left(\frac{7^4}{4^2}\right)$

(C) $\ln\left(\frac{7^4}{4^6}\right)$

(D) $\ln(20)$

9. Level: AP 5

$\int_1^\infty xe^{-x}dx =$

(A) $\frac{1}{2}e^2$ (B) $-\frac{1}{2}e^2$ (C) $\frac{2}{e}$ (D) The integral diverges.

Free Response Practice

1. Let $f(x)=\int_3^x \frac{1}{x^2+k}\,dx$, where k is a constant.

(a) Let $k=9$. Find $f\left(\sqrt{3}\right)$.

(b) Let $k=-1$. Find $f\left(4\right)$.

(c) Let $k=0$. Find $\lim_{x\to\infty} f\left(x\right)$ or show that the limit does not exist.

(d) Let $g\left(x\right)=f'\left(x\right)$. Given that $k\neq 0$, determine if g has a local minimum, a local maximum, or neither at $x=0$. Give a reason for your answer.

Additional Multiple-Choice Practice

10. Level: AP 4

$$\int \frac{1}{x^2-4x+20}\,dx =$$

(A) $\dfrac{1}{\frac{x^3}{3}-2x^2+20x}+C$

(B) $\ln\left|x^2+4x-21\right|+C$

(C) $\arcsin\left(\dfrac{x+2}{5}\right)+C$

(D) $\dfrac{1}{4}\arctan\left(\dfrac{x-2}{4}\right)+C$

11. Level: AP 4

Which of the following three integrals converge?

I. $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\,dx$ II. $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$ III. $\displaystyle\int_0^1 \frac{1}{x^4}\,dx$

(A) I only

(B) I and II only

(C) II and III only

(D) I, II, and III

12. Level: AP 3

$$\int \frac{3}{(x-1)(x-4)}\,dx =$$

(A) $\ln\left(\dfrac{x-4}{x-1}\right)+C$

(B) $3\ln\left(\dfrac{x-1}{x-4}\right)+C$

(C) $\ln(x-1)-\ln(x-4)+C$

(D) $\ln(x-4)+\ln(x-1)+C$

13. Level: AP 4

$$\int \frac{x^2}{x^2+1}\,dx =$$

(A) $\ln\left(x^2+1\right)+C$

(B) $x+\dfrac{x^3}{3}+C$

(C) $x-\tan^{-1}(x)+C$

(D) $\dfrac{x^3}{3}\tan^{-1}(x)+C$

14. Level: AP 3

$$\int \frac{x}{x^2+1}\,dx =$$

(A) $\ln\left(x^2+1\right)+C$

(B) $\dfrac{1}{2}\ln\left(x^2+1\right)+C$

(C) $2\ln\left(x^2+1\right)+C$

(D) $\dfrac{1}{2}\tan^{-1}(x)+C$

15. Level: AP 3

$$\int \frac{1}{x^2+1}\,dx =$$

(A) $\ln\left(x^2+1\right)+C$

(B) $\dfrac{\ln\left(x^2+1\right)}{2x}+C$

(C) $\tan^{-1}(x)+C$

(D) $\sin^{-1}(x)+C$

16. Level: AP 4

$$\int \frac{1}{x^2-1}\,dx =$$

(A) $\ln\left(\frac{x-1}{x+1}\right)+C$

(B) $\frac{1}{2}\ln\left(\frac{x-1}{x+1}\right)+C$

(C) $\frac{1}{2}\ln\left(x^2-1\right)+C$

(D) $\tan^{-1}(x)+C$

17. Level: AP 4

$$\int \frac{x}{x^2-1}\,dx =$$

(A) $\ln\left(x^2-1\right)+C$

(B) $\frac{1}{2}\ln\left(\frac{x-1}{x+1}\right)+C$

(C) $\frac{1}{2}\ln\left(x^2-1\right)+C$

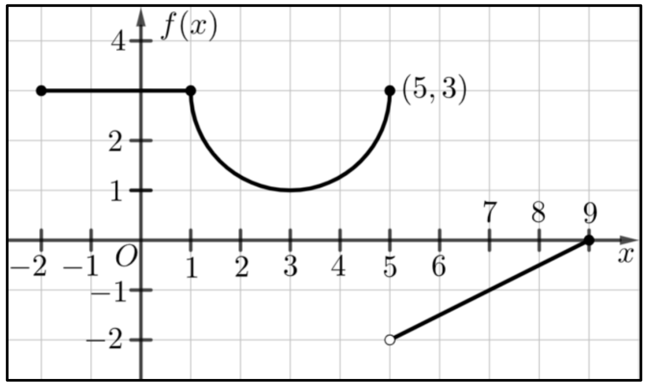
(D) $\ln(x)-\frac{x^2}{2}+C$

Additional Open Response Practice

18. Level: AP 5

Given $\int_{2A}^{3A} \frac{x+A}{x(x-A)}\,dx = \ln\left(\frac{A}{3}\right)$, find A.

Additional Free Response Practice



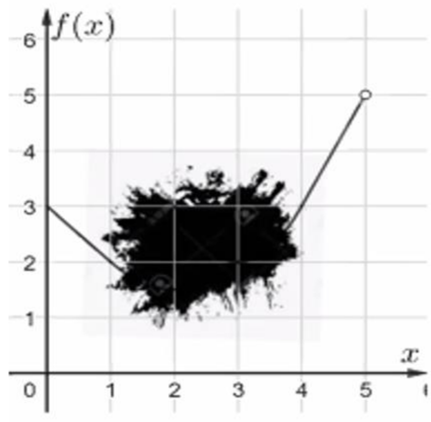
2. The function $f(x)$ is defined for $-2 \le x \le 9$ and consists of two line segments and a semi-circle as shown in the figure above.

(a) $\int_9^1 \left(f(x)+3\right)dx =$

(b) $\int_{-2}^0 xf'\left(x^2-1\right)dx =$

(c) $\int_0^3 xf'(x)\,dx =$

(d) Let g be a differentiable function where $g'(x) = f(x)$ and $g(7) = 2$. Find $g(5)$.



Graph created by Bryan Passwater

3. The function f is continuous and differentiable on the interval $0 \le x \le 5$. A portion of the graph of $f(x)$ is obscured by a coffee stain. It is known that $\int_0^4 f(x)\,dx = 8$ and $\int_4^3 f(x)\,dx = -3$ and $f(x)$ is linear on the intervals $(0,1)$ and $(4,5)$. For $x \ge 5$, $f(x) = \dfrac{4}{(x-a)^2}$, where a is a positive number.

(a) Find $\int_0^4 2xf'(x)\,dx$.

(b) Find $\int_1^2 x\,f(x^2-1)\,dx$.

(c) Find $\int_0^{\pi/2} \cos x\, f'(\sin x)\,dx$.

(d) It is known that $\int_3^{\infty} f(x)\,dx = 9$, find the value of a.