GENERAL INSTRUCTIONS

You may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as your answers. Correct answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.
- You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in standard mathematical notation rather than calculator syntax.

  For example, $\int_1^5 x^2 dx$ may not be written as fnInt (X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

PART A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

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1. Two particles travel in the xy-plane. For time $t \geq 0$, the position of particle A is given by $x = t + 1$ and $y = (t + 1)^2 - 2t - 2$, and the position of particle B is given by $x = 4t - 2$ and $y = -2t + 2$.

   (a) Find the velocity vector for each particle at time $t = 2$.

   (b) Set up an integral expression for the distance traveled by particle A from time $t = 1$ to $t = 3$. Do not evaluate the integral.

   (c) At what time do the two particles collide? Justify your answer.

   (d) Sketch the path of both particles from time $t = 0$ to $t = 4$. Indicate the direction of each particle along its path.

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2. Let f be the function given by $f(x) = 2x^4 - 4x^2 + 1$.

   (a) Find an equation of the line tangent to the graph at $(-2,17)$. Verify your answer.

   (b) Find the x and y-coordinates of the relative maxima and relative minima.

   (c) Find the x-coordinates of the points of inflection. Verify your answer.

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3. Water is draining at the rate of $48\pi \text{ft}^3/\text{sec}$ from the vertex at the bottom of a conical tank whose diameter at its base is 40 feet and whose height is 60 feet.

   (a) Find an expression for the volume of water (in ft^3) in the tank in terms of its radius at the surface of the water.

   (b) At what rate (in ft/sec) is the radius of the water in the tank shrinking when the radius is 16 feet?

   (c) How fast (in ft/sec) is the height of the water in the tank dropping at the instant that the radius is 16 feet?

PART B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

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4. Let f be the function given by $f(x) = e^{-4x^2}$

   (a) Find the first four non-zero terms and the general term of the power series for $f(x)$ about $x = 0$.

   (b) Find the interval of convergence of the power series for $f(x)$ about $x = 0$. Show the analysis that leads to your conclusion.

   (c) Use term-by-term differentiation to show that $f'(x) = -8xe^{-4x^2}$

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5. Let R be the region enclosed by the graphs of $y = 2\ \ln x$ and $y = \dfrac{x}{2}$, and the lines $x = 2$ and $x = 8$.

   (a) Find the area of R.

   (b) Set up, but <u>do not integrate</u>, an integral expression, in terms of a single variable, for the volume of the solid generated when R is revolved about the x-axis.

   (c) Set up, but <u>do not integrate</u>, an integral expression, in terms of a single variable, for the volume of the solid generated when R is revolved about the line $x = -1$.

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6. Let f and g be functions that are differentiable throughout their domains and that have the following properties:

(i) $f(x + y) = f(x)g(y) + g(x)f(y)$

(ii) $\lim_{a \to 0} f(a) = 0$

(iii) $\lim_{h \to 0} \frac{g(h) - 1}{h} = 0$

(iv) $f'(0) = 1$

(a) Use L'Hôpital's Rule to show that $\lim_{a \to 0} \frac{f(a)}{a} = 1$.

(b) Use the definition of the derivative to show that $f'(x) = g(x)$.

(c) Find $\int \frac{g(x)}{f(x)} dx$

END OF EXAMINATION