

CALCULUS BC

SECTION I, Part B

Time—50 Minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

**Directions:** Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

**In this test:**

1. The **exact** numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
2. Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

29. The first three non-zero terms in the Taylor series about  $x = 0$  for  $f(x) = \cos x$

(A)  $x + \frac{x^3}{3!} + \frac{x^5}{5!}$

(B)  $x - \frac{x^3}{3!} + \frac{x^5}{5!}$

(C)  $1 - \frac{x^2}{2!} + \frac{x^4}{4!}$

(D)  $1 - \frac{x^2}{2!} - \frac{x^4}{4!}$

(E)  $1 + \frac{x^2}{2!} + \frac{x^4}{4!}$

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30.  $\int \cos^3 x \, dx =$

(A)  $\frac{\cos^4 x}{4} + C$

(B)  $\frac{\sin^4 x}{4} + C$

(C)  $\sin x - \frac{\sin^3 x}{3} + C$

(D)  $\sin x + \frac{\sin^3 x}{3} + C$

(E)  $\sin^3 x + C$

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31. If  $f(x) = (3x)^{(3x)}$  then  $f'(x) =$

(A)  $(3x)^{(3x)}(3\ln(3x) + 3)$

(B)  $(3x)^{(3x)}(3\ln(3x) + 3x)$

(C)  $(9x)^{(3x)}(\ln(3x) + 1)$

(D)  $(3x)^{(3x-1)}(3x)$

(E)  $(3x)^{(3x-1)}(9x)$

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32. To what limit does the sequence  $S_n = \frac{3+n}{3^n}$  converge as  $n$  approaches infinity?

- (A) 1                      (B)  $\frac{1}{3}$                       (C) 0                      (D)  $\infty$                       (E) 3

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33.  $\int \frac{18x-17}{(2x-3)(x+1)} dx =$

- (A)  $8\ln|2x-3| + 7\ln|x+1| + C$   
(B)  $2\ln|2x-3| + 7\ln|x+1| + C$   
(C)  $4\ln|2x-3| + 7\ln|x+1| + C$   
(D)  $7\ln|2x-3| + 2\ln|x+1| + C$   
(E)  $\frac{7}{2}\ln|2x-3| + 4\ln|x+1| + C$

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34. A particle moves along a path described by  $x = \cos^3 t$  and  $y = \sin^3 t$ . The distance that the particle travels along the path from  $t = 0$  to  $t = \frac{\pi}{2}$  is

- (A) 0.75                      (B) 1.50                      (C) 0                      (D) -3.50                      (E) -0.75

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35. The sale price of an item is  $800 - 35x$  dollars and the total manufacturing cost is  $2x^3 - 140x^2 + 2,600x + 10,000$  dollars, where  $x$  is the number of items. What number of items should be manufactured in order to optimize the manufacturer's total profit?

- (A) 35                      (B) 25                      (C) 10                      (D) 15                      (E) 20
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36. The area enclosed by the polar equation  $r = 4 + \cos \theta$ , for  $0 \leq \theta \leq 2\pi$ , is

- (A) 0                      (B)  $\frac{9\pi}{2}$                       (C)  $18\pi$                       (D)  $\frac{33\pi}{2}$                       (E)  $\frac{33\pi}{4}$
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37. Use the trapezoid rule with  $n = 4$  to approximate the area between the curve  $y = x^3 - x^2$  and the  $x$ -axis from  $x = 3$  to  $x = 4$ .

- (A) 35.266                      (B) 27.766                      (C) 63.031                      (D) 31.516                      (E) 25.125
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38. If  $f(x) = \sum_{k=0}^{\infty} (\cos^2 x)^k$ , then  $f\left(\frac{\pi}{4}\right)$  is
- (A) -2                      (B) -1                      (C) 0                      (D) 1                      (E) 2
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39. The volume of the solid that results when the area between the graph of  $y = x^2 + 2$  and the graph of  $y = 10 - x^2$  from  $x = 0$  to  $x = 2$  is rotated around the  $x$ -axis is

- (A)  $2\pi \int_0^2 y(\sqrt{y-2}) dy + 2\pi \int_0^2 y(\sqrt{10-y}) dy$
- (B)  $2\pi \int_2^6 y(\sqrt{y-2}) dy + 2\pi \int_6^{10} y(\sqrt{10-y}) dy$
- (C)  $2\pi \int_2^6 y(\sqrt{y-2}) dy - 2\pi \int_6^{10} y(\sqrt{10-y}) dy$
- (D)  $2\pi \int_0^2 y(\sqrt{y-2}) dy - 2\pi \int_0^2 y(\sqrt{10-y}) dy$
- (E)  $2\pi \int_0^2 y(\sqrt{10-y}) dy - 2\pi \int_0^2 y(\sqrt{y-2}) dy$
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40.  $\int_0^4 \frac{dx}{\sqrt{9+x^2}} =$

- (A)  $\ln 3$                       (B)  $\ln 4$                       (C)  $-\ln 2$                       (D)  $-\ln 4$                       (E) Undefined
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41. The rate that an object cools is directly proportional to the difference between its temperature (in Kelvins) at that time and the surrounding temperature (in Kelvins). If an object is initially at  $35K$ , and the surrounding temperature remains constant at  $10K$ , it takes 5 minutes for the object to cool to  $25K$ . How long will it take for the object to cool to  $20K$ ?

- (A) 6.66 min.      (B) 7.50 min.      (C) 7.52 min.      (D) 8.97 min.      (E) 10.00 min.
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42.  $\int e^x \cos x \, dx =$

(A)  $\frac{e^x}{2}(\sin x + \cos x) + C$

(B)  $\frac{e^x}{2}(\sin x - \cos x) + C$

(C)  $\frac{e^x}{2}(\cos x - \sin x) + C$

(D)  $2e^x(\sin x + \cos x) + C$

(E)  $e^x(\sin x + \cos x) + C$

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43. Two particles leave the origin at the same time and move along the  $y$ -axis with their respective positions determined by the functions  $y_1 = \cos 2t$  and  $y_2 = 4 \sin t$  for  $0 < t < 6$ . For how many values of  $t$  do the particles have the same acceleration?

- (A) 0      (B) 1      (C) 2      (D) 3      (E) 4
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44. The minimum value of the function  $y = x^3 - 7x + 11$ ,  $x \geq 0$ , is approximately
- (A) 18.128      (B) 9.283      (C) 6.698      (D) 5.513      (E) 3.872
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45. Use Euler's Method with  $h = 0.2$  to estimate  $y(1)$ , if  $y' = y$  and  $y(0) = 1$ .
- (A) 1.200      (B) 2.075      (C) 2.488      (D) 4.838      (E) 9.677
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**STOP**

**END OF SECTION I**

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

