Chapter 4: The Derivative in Graphing and Applications

Summary: The main purpose of this chapter is to use the derivative as a tool to assist in the graphing of functions and for solving optimization problems. The most prominent use of the derivative is to help determine the overall behavior of a function. Types of behavior that can be described by using the derivative are when the function is increasing or decreasing and where the function is concave up or down. From these features, conclusions can be obtained about the extrema and inflection points of a function.

This chapter, then, intends to show some of the many uses of derivatives. For example, derivatives can be used to find the roots of functions. This is the primary motivation behind Newton's Method. In motion problems where a position function describes the location of an object or a particle, the derivatives of the position function have special meanings such as the velocity and acceleration of the particle. Derivatives also find their way into many application problems where certain quantities either need to be maximized or minimized. Because derivatives can be used to find the extreme points of a function, they can be instrumental in optimizing quantities in many application problems.

OBJECTIVES: After reading and working through this chapter you should be able to do the following:

1. Determine where a function is increasing or decreasing (§4.1, 4.2).
2. Determine the concavity of a function (§4.1).
3. Locate critical points (§4.2), relative extrema (§4.2), and points of inflection of a function (§4.1).
4. Sketch the curve of a function based upon information from its derivatives together with information about asymptotes and intercepts (§4.2, 4.3).
5. Find absolute extrema of a continuous function on a closed interval (§4.4).
6. Find the maximizing or minimizing value in various application problems (§4.5).

7. Use derivatives to discuss the motion of a particle that has a position function for its location along a line (§4.6).
8. Apply Newton's Method to find roots of functions (§4.7).
9. Draw conclusions about the value of a functions derivative at a point by using the Mean-Value Theorem and Rolle's Theorem (§4.8).

4.1 Analysis of Functions I: Increase, Decrease, and Concavity

PURPOSE: To relate the derivative of a function to the ideas of increasing, decreasing and concavity of functions.

The primary focus of this section is to introduce the ideas of the increase and decrease of functions, the concavity of functions and how all of these relate to the derivative of a function. As it turns out, there are some convenient ways to remember these concepts that depend upon the tangent line of a function at any given point. Since the derivative can be used to find the slope of a tangent line, it becomes an important method for discussing the increase, decrease and concavity of a function.

increasing and decreasing

Increasing and **decreasing** describe whether the values of the function are getting larger or smaller as the inputs increase. The tangent line of a function can help keep all of this in the right perspective. A curve will follow the direction of its tangent line at any given point. So if a tangent line has a positive slope, then the function should be increasing at that point. Since the derivative is what allows us to determine the slope of the tangent line, it can help determine whether a function is increasing or decreasing.

IDEA: Increasing functions have a positive derivative and decreasing functions have a negative derivative.

To summarize, increasing functions have a derivative that is positive, while decreasing functions have a derivative that is negative. Of course, this is all assuming that the function being discussed is differentiable. If it is not differentiable then these assertions cannot be made.

concavity

"if it's a frown, it's down"

Concavity is the other big topic. Simply put, concavity describes how the curve is bending. If it is bending upward in a U shape then the concavity of the function is positive. If it is bending downward then the concavity of the function is negative. The phrase **"if it's a frown, it's down"** is meant to describe the concavity. If the shape of a curve makes a frown (i.e., it is bending downward) then the concavity of the curve is negative (or downward).

IDEA: Concavity describes whether the derivative is increasing or decreasing.

What concavity is really describing is when the derivative of a function is increasing or decreasing. If a function has a derivative that is increasing, then it will be bending upward because its slopes will be gradually becoming more positive. Unfortunately, the sign of the derivative of f does not provide curvature direction. The second derivative is usually related to concavity in the way that the first derivative is related to increase and decrease of a function. If $f'' > 0$ then the function f will have a concave upward shape while if $f'' < 0$ then the function f will have a concave downward shape.

One interesting point that can be identified on a curve is called **a point of inflection**. This is a point on the curve where the bending of the curve changes directions. In other words, if the concavity changes from positive to negative or vice versa then the point where this happens is a point of inflection. Similar points can be identified where a function changes from increasing to decreasing or vice versa although these points are not called inflection points. They are described in more detail in the next section.

point of inflection

Determining where a function is increasing or decreasing, and how its concavity is behaving can allow a rough sketch of a function to be drawn. This is where the **analysis of the signs** of the derivatives becomes important. By knowing where the first derivative is positive or negative, the increase or decrease of a function can be found. Similarly, by knowing where the second derivative is positive or negative can determine the concavity of a function.

sign analysis

IDEA: Finding where derivatives equal zero or are undefined can help determine intervals of increase or decrease and concavity.

Sign analysis first involves finding locations where the derivatives are zero or undefined. These points are used to determine intervals on which to check whether each derivative is positive or negative. Then conclusions can be made about the behavior of the function.

One important thing to note about the zeros of the derivatives is that simply because $f'(a) = 0$ does not mean that $f(x)$ will change from increasing to decreasing at $x = a$. For example, consider $y = x^3$. Since $y' = 3x^2$, then $y' = 0$ when $x = 0$. However, the function $y = x^3$ is increasing for all x-values. Similarly, even though $f''(a) = 0$, it does not mean that there must be a point of inflection at $x = a$. For example, consider $y = x^4$. Since $y'' = 12x^2$ then $y'' = 0$ when $x = 0$. However, the function $y = x^4$ is concave upwards for all x-values meaning that there is not a point of inflection at $x = 0$.

Checklist of Key Ideas:

- ☐ increasing and decreasing functions
- ☐ constant functions
- ☐ critical numbers and relative extrema
- ☐ concavity
- ☐ inflection points
- ☐ sign analysis

4.2 Analysis of Functions II: Relative Extrema; Graphing Polynomials

PURPOSE: To use the first and second derivative tests to find relative extrema and to sketch polynomials.

In this section, the analysis of derivatives are used to find relative or local extrema of a function. Two tests are introduced: the first derivative test and the second derivative test. First, it is noticed that if a relative extrema occurs on a continuous function then it is required that the derivative of the function at the extrema must either be equal to zero or not defined at that point. The list of all points where the derivative is undefined or equal to zero are called **critical points** (**stationary points** are simply critical points where the derivative is defined and equal to zero). Then the basic process for graphing a function includes finding all of the critical points of the function since these are the locations where the function may have relative extrema.

critical points
stationary points

Think of critical points as all of the possible places that we may have a relative extrema. Once the critical points have been determined the first derivative and second derivative tests may be applied to determine if there is a relative extrema or not.

IDEA: The first derivative test finds relative extrema based upon changes in the sign of the first derivative.

first derivative test

A relative extrema indicates that a function is defined at that point and that the function must change from increasing to decreasing at that point (or vice versa). This is essentially what the **first derivative test** does; it is a sign analysis of the first derivative. If the derivative changes signs at the critical point then there is a relative extrema located there. If the function changes from increasing to decreasing (from up to down) then the point is a relative maximum. Changing from decreasing to increasing (the sign of the derivative would change from negative to positive) would indicate a relative minimum.

IDEA: The second derivative test finds relative extrema based upon the sign of $f''(x)$ at critical points.

second derivative test

The **second derivative test** goes one step further. If there is a critical point then a relative maximum can only occur when the concavity is negative at that point. This is essentially the second derivative test. If the second derivative is positive at a critical point (concavity is positive) then there is a relative minimum. On the other hand if the second derivative is negative at a critical point then there is a relative maximum at that point.

sketching polynomials

Polynomials are relatively easy to analyze since their derivatives are easy to find. Items of interest for polynomials are x-intercepts (roots), y-intercepts, intervals of increase/decrease, intervals of concavity, critical points, relative extrema, points of inflection, and end behavior. By putting all of these things together, a **sketch of the polynomial** can be easily obtained.

To **start the sketch**, it may be easiest to first find the interesting points on the curve such as intercepts and critical points. The multiplicity of roots can also be used to find out information about the function. Generally, if a root has a multiplicity of greater than one, then the function will have a critical point at that root. If the multiplicity is even at that root, then it will be a relative extrema. Then sign analysis can also give the general shape of the curve.

Starting a sketch

1. Find interesting points
   → roots/intercepts
   → critical points
   → inflection points
2. determine the shape
   → increasing or decreasing?
   → concavity?

CAUTION: Several of the techniques listed here may give redundant information but there should not be any contradictions. If a contradiction is found then there may be an error in some calculation.

Checklist of Key Ideas:

- ☐ relative maxima and minima
- ☐ critical points
- ☐ stationary points
- ☐ First Derivative Test
- ☐ Second Derivative Test
- ☐ multiplicity of roots
- ☐ graphing polynomials
- ☐ properties of polynomials

4.3 Analysis of Functions III: Rational Functions, Cusps, and Vertical Tangents

PURPOSE: To graph more challenging functions.

Sketching rational functions is very similar to sketching polynomial functions. All of the same information can be used. The one thing to be cautious about is that some rational functions may have vertical asymptotes. These are places of discontinuity which are interesting but they are not critical points (since the function is not defined at these points). But when determining intervals of increase/decrease and concavity, the points of discontinuity should be listed as possible endpoints of intervals.

In addition to intercepts, critical points, points of inflection, and intervals of increase/decrease and concavity, rational functions will also potentially have vertical and/or horizontal asymptotes. **Vertical asymptotes** may be determined by factors in the denominator which do not cancel with the numerator (if they cancel completely then there is a hole at the zero of the factor). If a factor in the denominator does not cancel then there will be a vertical asymptote at the zero of the factor.

vertical asymptotes
(see also §1.3)

horizontal asymptotes (see also §1.3)

Horizontal asymptotes may only occur when the degree of the numerator is equal to or less than the degree of the denominator. The position of any horizontal asymptotes may then by found by taking limits as $x \to \pm\infty$ of the leading terms of the numerator and the denominator. These limits may sometimes be calculated using L'Hôpital's rule (see §3.5).

Checklist of Key Ideas:

- ☐ graphing rational functions
- ☐ symmetries
- ☐ intercepts
- ☐ periodicity
- ☐ relative extrema
- ☐ concavity
- ☐ intervals of increase and decrease
- ☐ inflection points
- ☐ asymptotes
- ☐ vertical tangents
- ☐ cusps

4.4 Absolute Maxima and Minima

PURPOSE: To locate the absolute extrema of a function.

In this section, information about relative extrema and the derivatives of a function can be used to determine if a function may have an absolute maximum or minimum. In general, there is no guarantee that a function will actually have an absolute maximum or minimum on a given interval. However, if certain things are known about both the function and the interval then there may be guarantees about finding the maxima and minima of the function upon the interval in question.

IDEA: If a function is continuous on a closed interval then the absolute extrema of the function can be found.

closed interval

For example, if a continuous function is on a **closed interval**, then there is guaranteed to be an absolute maximum and minimum of the function (the highest and lowest points on the interval). These points have to occur either at the endpoints or at any critical points in the interior of the interval. This guarantee immediately disappears if the function is discontinuous or the interval is not finite or closed.

IDEA: If absolute extrema exist, they will occur at the endpoints of an interval or at critical points.

On an **infinite interval**, absolute extrema can sometimes be found depending upon the end behavior of a function. For example, if $\lim_{x \to \pm\infty} f(x) = \infty$, the function will have an absolute minimum if it is continuous. The absolute minimum will have to occur at one of the functions relative minimum. A similar statement can be made about an absolute maximum if the functions end behavior approaches $-\infty$ all the time.

infinite interval

Open intervals or half-open intervals can cause problems and there are no guarantees. Each situation may be different. If there are going to be any absolute maximums or minimums on an open interval, however, they will have to occur either at the closed endpoints (if there are any) or the relative extrema of the function (if there are any). A simple example can show the problems that can arise. The function $y = x$ on the interval $(0,1)$ has no absolute maximum or minimum. The highest value approaches $y = 1$ but it never gets there. Likewise with the lowest value and $y = 0$. To determine if there are absolute extrema, compare the limits at the open endpoints with the function values at the critical points of the function.

open interval

Checklist of Key Ideas:

- ☐ absolute extrema
- ☐ extreme value theorem
- ☐ closed interval
- ☐ open interval
- ☐ infinite interval

4.5 Applied Maximum and Minimum Problems

PURPOSE: To solve optimization problems by using the techniques of this chapter.

Optimization problems or applied maximum and minimum problems, follow from the methods discussed earlier in this chapter. The approach to finding maximum and minimum values of a function in an applied setting is the same as earlier in this chapter.

In an applied problem, the first order of business is **determining the variables involved** and the equations that need to be considered. There may often be more than one equation. In these cases, one variable should be solved for in terms of the other(s) and substituted into the appropriate equations. The goal is to reduce all of the equations to a **single equation** with only one independent variable. Anything else cannot be considered using the techniques presented here. Sometimes drawing pictures and using geometric information may be helpful in reducing the number of equations.

determine the variables involved

obtain a single equation
→ geometry?
→ draw a picture?

determine the interval

After a single equation is obtained with one variable, the **interval** over which the optimization needs to occur has to be **determined**. This can usually be determined by statements in the problem. For example, if one of the variables represents a physical quantity like the height of a boy, then it cannot have any negative values. That information may help to determine the interval of the function.

what is to be found?

Finding Maximums/Minimums

→ identify variables

→ get one equation

→ identify interval

→ find critical points

→ behavior at endpoints and critical points

Finally, it should be determined **what is needed (i.e., a maximum or minimum)**. Then taking derivatives begins so that the function can be analyzed using the techniques of this chapter. First find critical points, then apply the First and Second derivative tests to find local extrema. Finally, determine which point of interest satisfies the problem statement. The values of other variables may need to be determined at the end (especially if there were more than one equation at the start with more than one variable).

Here are the general steps: (1) list the variables, (2) draw a picture if necessary, (3) list equations, (4) reduce to one equation with one variable, (5) determine the appropriate interval, (6) analyze the function by taking derivatives, applying derivative tests, etc., (7) find all relevant information.

Checklist of Key Ideas:

- ☐ optimization over finite closed interval
- ☐ optimization over infinite interval or open finite interval
- ☐ steps for solving optimization problems
- ☐ economics application

4.6 Rectilinear Motion

PURPOSE: To discuss motion along a straight line.

position and velocity
$v(t) = s'(t)$

The convention with rectilinear motion (or motion along a line) is that movement to the right is in the positive direction and movement to the left is in the negative direction. The sign of the **position function**, $s(t)$, determines on which side of the origin a particle is located. The sign of the **velocity function**, $v(t)$, determines in which direction a particle is moving. A positive value for $v(t)$ indicates movement to the right, for example. If $v(t) = 0$, then the particle is momentarily stationary.

speed = $|v(t)|$

speed is always nonnegative

Speed is often confused with velocity. Speed only indicates how large the velocity is (in an absolute value sense ignoring any positive or negative signs). A velocity of $+50$ m/s and -50 m/s both have a speed of 50 meters per second. Because **speed ignores negative signs**, it is possible that the speed may be increasing while the velocity is decreasing.

IDEA: Speed tells how fast an object is moving but does not tell what direction it is moving.

A particle with rectilinear motion is **speeding up** if its velocity and acceleration both have the same sign. If the velocity and acceleration both have opposite signs, then the particle is **slowing down** (approaching a velocity of zero). Another way to interpret this is that speed only describes how fast an object is moving but not what direction. If a particle is slowing down, then its velocity will be close to zero. If the particle is speeding up, then its velocity is moving away from zero (either in a positive or negative sense).

speeding up → speed increases
slowing down → speed decreases

Checklist of Key Ideas:

- ☐ rectilinear motion
- ☐ position
- ☐ displacement
- ☐ velocity
- ☐ speed
- ☐ acceleration

4.7 Newton's Method

PURPOSE: To find the roots of a function using Newton's Method.

This section builds off of the idea that the derivative can be used to find a **linear approximation** of a function near a point that was first discussed in Chapter 3. This linear approximation is just a tangent line to the function. The idea in this section is that if a tangent line is a good approximation of a function, then finding the zero of a tangent line may help to approximate the zero of a function. This is the idea that is called Newton's Method.

linear approximation
(see §3.5)

IDEA: Newton's Method uses a sequence of tangent lines to approximate the root of a function.

If a tangent line to a function $y = f(x)$ at the point x_0 is given by $y = y_0 + m_0(x - x_0)$ then it is a straightforward process to find the zero of this line. For example, if $y = 0$ then $x = x_0 - y_0/m_0$. But this is exactly the formula for the **iteration of Newton's method** when $y_0 = f(x_0)$ and $m_0 = f'(x_0)$.

$$x_{n+1} = x_n - y_n/m_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

Newton's method iteration
x_0 is original guess of root
x_n is the n-th root approximation

Newton's method is repeating this process of finding the zeros for tangent lines over and over again. A tangent line is constructed for the function at the location of each zero that was found in the previous step. Eventually, the zeros should approach the value of a zero of the function.

try to pick x_0 close to a root

There are some drawbacks to this method. Since the basic idea is to use a linear approximation (i.e., the tangent line), there will be problems when trying to approximate the function farther away from the point of tangency. What does this mean? Linear approximations only work well within a small area. The moral of all of this is that **a poorly chosen starting point can cause the method to fail**.

horizontal tangents can cause failure

Another way that **the method can fail is if the tangent line at any given step is horizontal**. The formula shown above, then has us dividing by zero which will not lead to any good result. Visually this is just reaffirming the fact that horizontal lines are parallel to the x-axis and will not have roots since they will never intersect the x-axis.

failure can occur if x_n and x_{n+1} have the same tangent line equation

One last way that Newton's method can fail is if the starting point leads to a new point that returns us to the original starting point. This may happen for example if **two consecutive points x_n and x_{n+1} both have the same equation for a tangent line**. In this type of situation, iteration will never stop. This problem can arise in even simple problems. For example, $y = x(x^2 - 20)$ with a starting point of $x_0 = 2$. The iterations will bounce back and forth between 2 and -2. To correct this problem, simply try a different starting point.

Checklist of Key Ideas:

- ☐ Newton's Method
- ☐ root finding
- ☐ linear approximation
- ☐ tangent line approximation
- ☐ pitfalls with Newton's Method

4.8 Rolle's Theorem; Mean-Value Theorem

PURPOSE: To introduce two theorems that relate to tangent lines.

This section states two important theorems: Rolle's Theorem and the Mean-Value Theorem. The Mean-Value Theorem is actually a more general case of Rolle's Theorem. Essentially Rolle's Theorem states a result about the existence of horizontal tangent lines for a function on a particular interval. In short, if the function values of a differentiable function are the same at the ends of an interval, then there will be a horizontal tangent line somewhere within the interval.

ROLLE'S THEOREM: If $f(a) = f(b)$ then there is a horizontal tangent line between $x = a$ and $x = b$ (if f is differentiable).

Rolle's Theorem

Here is another version of **Rolle's Theorem**: if the graph of a differentiable function intersects the x-axis at two places $x = a$ and $x = b$, then somewhere between

$x = a$ and $x = b$ there will be at least one point, $x = c$, where there is a horizontal tangent line. The value of c does not have to be unique. For example, the function $y = x^4 - 8x^2 - 9 = (x^2+1)(x^2-9)$ has zeros along the x-axis at $x = -3$ and $x = 3$. However, when Rolle's Theorem is applied to the zeros at $x = -3$ and $x = 3$, there are three places where this function has horizontal tangents in between these two points.

Rolle's Theorem is a specific application of the Mean-Value Theorem. The **Mean-Value Theorem** essentially says that on a particular interval there will be a tangent line that is parallel to the secant line through the function values at the endpoints of the interval. In the case of Rolle's Theorem, this is a horizontal line.

Mean-Value Theorem

MEAN-VALUE THEOREM (MVT): There is a tangent line between $x = a$ and $x = b$ that is parallel to the line between the points on $f(x)$ at $x = a$ and $x = b$ (if f is differentiable).

Here is a restatement of the Mean-Value Theorem (MVT): between any two points, $A\ (a, f(a))$ and $B\ (b, f(b))$, on the graph of a differentiable function $y = f(x)$, there is at least one point where the tangent line to the graph is parallel to the secant line that joins the points A and B.

Here is a another restatement of the MVT: over the interval $[a,b]$, the instantaneous velocity of a differentiable position function must equal the average velocity at least once.

The **Constant Difference Theorem** says, "If f and g have the same derivative on an open interval, then the graphs of f and g are vertical translations of one another over that interval." In other words, two functions that have the same derivative must differ from each other by a constant value.

Constant Difference Theorem

IDEA:A function must be differentiable for these theorems to be applied.

The main thing to watch out for with each of these theorems is that the function must be differentiable over the entire interval that is being discussed. Discontinuous functions cause problems with these theorems from the start since not being continuous immediately indicates that the function cannot be differentiable. For example, no conclusions about the function $y = 1/x$ can be drawn over any interval that contains $x = 0$ since it is discontinuous at that point.

Checklist of Key Ideas:

- ☐ Mean-Value Theorem
- ☐ Rolle's Theorem
- ☐ velocity
- ☐ constant difference theorem

Chapter 4 Sample Tests

Section 4.1

1. Answer true or false. If $f'(x) > 0$ for all x on the interval I, then $f(x)$ is concave up on the interval I.
2. Answer true or false. A point of inflection always has an x-coordinate where $f''(x) = 0$.
3. The largest interval over which f is increasing for $f(x) = (x-5)^4$ is
   (a) $[5, \infty)$
   (b) $[-5, \infty)$
   (c) $(-\infty, 5]$
   (d) $(-\infty, -5]$
4. The largest interval over which f is decreasing for $f(x) = x^3 - 12x + 7$ is
   (a) $(-\infty, -2]$
   (b) $[2, \infty)$
   (c) $[-2, 2]$
   (d) $[-2, \infty)$
5. The largest interval over which f is increasing for $f(x) = \sqrt[3]{x-2}$ is
   (a) $[2, \infty)$
   (b) $(-\infty, 2)$
   (c) $(-\infty, \infty)$
   (d) nowhere
6. The largest open interval over which f is concave up for $f(x) = \sqrt[3]{x-5}$ is
   (a) $(-\infty, 5)$
   (b) $(5, \infty)$
   (c) $(-\infty, \infty)$
   (d) nowhere
7. The largest open interval over which f is concave up for $f(x) = e^{x^4}$ is
   (a) $(-\infty, 0)$
   (b) $(0, \infty)$
   (c) $(-\infty, \infty)$
   (d) nowhere
8. The function $f(x) = x^{4/5}$ has a point of inflection with an x-coordinate of
   (a) 0
   (b) $4/5$
   (c) $-4/5$
   (d) None exist.
9. The function $f(x) = e^{x^6}$ has a point of inflection with an x-coordinate of
   (a) $-e$
   (b) e
   (c) 0
   (d) None exist.
10. Use a graphing utility to determine where $f(x) = \sin x$ is decreasing on $[0, 2\pi]$.
    (a) $[0, \pi]$
    (b) $[\pi, 2\pi]$
    (c) $[\pi/2, 3\pi/2]$
    (d) $[0, 2\pi]$
11. Answer true or false. $y = \cot x$ has a point of inflection on the interval $(0, \pi)$.
12. Answer true or false. All functions of the form $f(x) = ax^n$ have an inflection point.
13. The function $f(x) = x^4 - 24x^2 + 6$ is concave down over which of the following intervals?
    (a) $(-\infty, 0) \cup (0, \infty)$
    (b) $(-\infty, 0) \cup (\sqrt{12}, \infty)$
    (c) $(-\infty, -2]$
    (d) $(-2, 2)$
14. Answer true or false. If $f''(-2) = -3$ and $f''(2) = 3$, then there must be a point of inflection on $(-2, 2)$.
15. The function $f(x) = \dfrac{x^2}{x^2 - 4}$ has
    (a) points of inflection at $x = -4$ and $x = 4$.
    (b) points of inflection at $x = -2$ and $x = 2$.
    (c) a point of inflection at $x = 0$.
    (d) no points of inflection.

Section 4.2

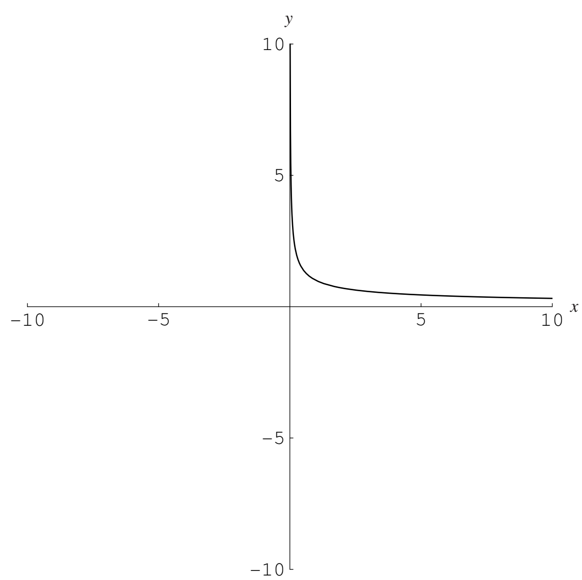
1. Determine the x-coordinate(s) for any stationary point(s) of the function $f(x) = 2x^3 - 3x^2 - 72x + 6$.
   (a) $x = -4$ and $x = 3$
   (b) $x = -3$ and $x = 4$
   (c) $x = -6$
   (d) No stationary points.
2. Determine the x-coordinate(s) of the critical point(s) of the function $f(x) = \sqrt[5]{x-3}$.
   (a) $x = 0$
   (b) $x = 3$
   (c) $x = -3$
   (d) No critical points.
3. Answer true or false. $f(x) = x^{2/7}$ has a critical point.
4. Answer true or false. A function has a relative extrema at every critical point.
5. The function $f(x) = x^2 + 6x + 8$ has a
   (a) relative maximum at $x = -3$.
   (b) relative minimum at $x = -3$.
   (c) relative maximum at $x = 3$.
   (d) relative minimum at $x = 3$.
6. The function $f(x) = \cos^2 x$ on $0 < x < 2\pi$ has
   (a) a relative maximum at $x = \pi$; relative minima at $x = \pi/2$ and $x = 3\pi/2$
   (b) relative maxima at $x = \pi/2$ and $x = 3\pi/2$; a relative minimum at $x = \pi$
   (c) a relative maximum at $x = \pi$; no relative minima
   (d) no relative maxima; a relative minimum at $x = \pi$.
7. The function $f(x) = x^4 - 4x^3$ has
   (a) a relative maximum at $x = 0$; no relative minima
   (b) no relative maxima; a relative minimum at $x = 3$
   (c) a relative maximum at $x = 0$; a relative minima at $x = 3$
   (d) a relative maximum at $x = 0$; relative minima at $x = -3$ and $x = 3$
8. Answer true or false. $f(x) = |\tan x|$ has no relative extrema on $(-\pi/2, \pi/2)$.
9. $f(x) = e^{2x}$ has
   (a) a relative maximum at $x = 0$
   (b) a relative minimum at $x = 0$
   (c) a relative minimum at $x = 2$
   (d) no relative extrema
10. $f(x) = |x^2 - 9|$ has
    (a) no relative maxima; a relative minimum at $x = 3$
    (b) a relative maximum at $x = 3$; no relative minima
    (c) relative minima at $x = -3$ and $x = 3$; a relative maximum at $x = 0$
    (d) relative minima at $x = -9$ and $x = 9$; a relative maximum at $x = 0$
11. $f(x) = \ln(x^2 + 2)$ has
    (a) a relative maximum only
    (b) a relative minimum only
    (c) both a relative maximum and minimum
    (d) no relative extrema
12. On the interval $(0, 2\pi)$, the function $f(x) = \sin x \cos(2x)$ has
    (a) a relative maximum only
    (b) a relative minimum only
    (c) both a relative maximum and a minimum
    (d) no relative extrema
13. Answer true or false. The function $f(x) = e^x \ln x$ has a relative minimum on $(0, \infty)$.
14. Answer true or false. A graphing utility can be used to show that $f(x) = |x|$ has a relative minimum.
15. Answer true or false. A graphing utility can be used to show that $f(x) = x^4 - 3x^2 + 3$ has two relative minima on $[-10, 10]$.

Section 4.3

1. Answer true or false. If $f'(-2) = -1$ and $f'(2) = 1$, then there must be a relative minimum on $(-2, 2)$.
2. The polynomial function $y = x^2 - 6x + 8$ has
   (a) one stationary point at $x = 3$.
   (b) two stationary points, one at $x = 0$ and one at $x = 3$.
   (c) one stationary point that is at $x = -3$.
   (d) one stationary point that is at $x = 0$.
3. The rational function $y = \dfrac{3x+6}{x^2-1}$ has
   (a) a horizontal asymptote at $y = 0$.
   (b) a horizontal asymptote at $y = -2$.
   (c) horizontal asymptotes at $x = -1$ and $x = 1$.
   (d) no horizontal asymptotes.
4. Determine the x-coordinates of all the stationary points of the rational function $y = \dfrac{3x+6}{x^2-1}$.

(a) $x = -2 \pm \sqrt{3}$
(b) $x = -2$ only
(c) both $x = -1$ and $x = 1$
(d) $x = -2, x = -1$ and $x = 1$

5. Answer true or false. The rational function $y = x^3 - \dfrac{1}{x^2}$ has no vertical asymptotes.
6. On a $[-10, 10]$ by $[-10, 10]$ window on a graphing utility the rational function $y = \dfrac{x^3+8}{x^3-8}$ can be determined to have
   (a) one horizontal asymptote and no vertical asymptote.
   (b) no horizontal asymptotes and one vertical asymptote.
   (c) one horizontal asymptote and one vertical asymptote.
   (d) one horizontal asymptote and three vertical asymptotes.
7. Use a graphing utility to graph $f(x) = x^{1/7}$. How many points of inflection does the function have?
   (a) 0
   (b) 1
   (c) 2
   (d) 3
8. Use a graphing utility to graph $f(x) = x^{-1/7}$. How many points of inflection are there?
   (a) 0
   (b) 1
   (c) 2
   (d) 3
9. Determine which function is graphed below.



   (a) $f(x) = x^{1/2}$
   (b) $f(x) = x^{-1/3}$
   (c) $f(x) = x^{-1/2}$
   (d) $f(x) = x^{1/3}$
10. Use a graphing utility to generate the graph of $f(x) = x^2 e^{3x}$, then determine the x-coordinates of all relative extrema on $(-10, 10)$ and identify them as relative maxima or minima.
   (a) There is a relative maximum at $x = 0$ and a relative minimum at $x = -2/3$.
   (b) There is a relative minimum at $x = 0$ and a relative maximum at $x = -2/3$.
   (c) There is a relative minimum at $x = 0$ and relative maxima at $x = -1$ and $x = 1$.
   (d) There are no relative extrema.
11. Answer true or false. Using a graphing utility, it can be shown that $f(x) = x^2 \tan x$ has a maximum on $0 < x < 2\pi$.
12. Answer true or false. $\lim_{x \to 0^+} \left(\sqrt{x} \ln x\right) = 0$.
13. Evaluate the limit: $\lim_{x \to 0^+} x^{3/2} \ln x =$
   (a) 0
   (b) 1
   (c) ∞
   (d) The limit does not exist.
14. Answer true or false. A fence is used to enclose a rectangular plot of land. If there are 160 feet of fencing, it can be shown that 40 ft by 40 ft square is the rectangle that can be enclosed with the greatest area. (A square is considered a rectangle).
15. Answer true or false. The function $f(x) = \dfrac{x^2+3x+4}{x-1}$ has an oblique asymptote.

Section 4.4

1. In the interval $[-2, 2]$, the function $f(x) = 3x^2 - x + 2$ has an absolute maximum value of
   (a) 16
   (b) 2
   (c) 12
   (d) 4
2. The function $f(x) = |5 - 2x|$ has an absolute minimum value of
   (a) 0

(b) 3

(c) 1

(d) 5

3. Answer true or false. $f(x) = x^3 - x^2 + 2$ has an absolute maximum and an absolute minimum.

4. Answer true or false. $f(x) = x^3 - 18x^2 + 20x + 2$ restricted to the domain of $[0, 20]$ has an absolute maximum at $x = 2$ of -22, and an absolute minimum at $x = 10$ of -598.

5. The function $f(x) = \sqrt{x-2}$ has an absolute minimum value of

   (a) 0 at $x = 2$.

   (b) 0 at $x = 0$.

   (c) -2 at $x = 0$.

   (d) 0 at $x = -2$

6. The function $f(x) = \sqrt{x^2+5}$ has an absolute maximum, if one exists, at

   (a) $x = -5$

   (b) $x = 5$

   (c) $x = 0$

   (d) No maximum exists.

7. Find the location of the absolute maximum of $y = \tan x$ on $[0, \pi]$, if it exists.

   (a) $x = 0$

   (b) $x = \pi$

   (c) $x = \pi/2$

   (d) No maximum exists.

8. $f(x) = x^2 - 3x + 2$ on $(-\infty, \infty)$ has

   (a) only an absolute maximum.

   (b) only an absolute minimum.

   (c) both an absolute maximum and minimum.

   (d) neither an absolute maximum nor an absolute minimum.

9. $f(x) = \dfrac{1}{x^2}$ on $[1, 3]$ has

   (a) an absolute maximum at $x = 1$ and an absolute minimum at $x = 3$.

   (b) an absolute minimum at $x = 1$ and an absolute maximum at $x = 3$.

   (c) no absolute extrema.

   (d) an absolute minimum at $x = 2$ and absolute maxima at $x = 1$ and $x = 3$.

10. Answer true or false. $f(x) = \sin x \cos x$ on $[0, \pi]$ has an absolute maximum at $x = \pi/2$.

11. Use a graphing utility to assist in determining the location of the absolute maximum of the function $f(x) = -(x^2 - 3)^2$ on $(-\infty, \infty)$, if it exists.

    (a) $x = \sqrt{3}$ and $x = -\sqrt{3}$

    (b) $x = \sqrt{3}$ only

    (c) $x = 0$

    (d) No absolute maximum exists.

12. Answer true or false. If $y = f(x)$ has an absolute minimum at $x = 2$, then $y = -f(x)$ also has an absolute minimum at $x = 2$.

13. Answer true or false. Every function has an absolute maximum and an absolute minimum if its domain is restricted to where f is defined on the interval $[-a, a]$, where a is finite.

14. Use a graphing utility to locate the value of x where $f(x) = x^4 - 3x + 2$ has an absolute minimum, if it exists.

    (a) $x = 1$

    (b) $x = \sqrt[3]{3/4}$

    (c) $x = 0$

    (d) No absolute minimum exists.

15. Use a graphing utility to estimate the absolute maximum value of $f(x) = (x-5)^2$ on $[0, 6]$, if it exists.

    (a) 25

    (b) 0

    (c) 1

    (d) No maximum exists.

Section 4.5

1. Express the number 20 as the sum of two nonnegative numbers, a and b (with $a \leq b$), whose product is as large as possible.

   (a) $a = 5, b = 15$

   (b) $a = 1, b = 19$

   (c) $a = 10, b = 10$

   (d) $a = 0, b = 20$

2. A right triangle has a perimeter of 16. What are the lengths of each side if the area contained within the triangle is to be maximized?

   (a) $16/3$, $16/3$, $16/3$

   (b) 5, 5, 6

   (c) $16 - 8\sqrt{2}$, $16 - 8\sqrt{2}$, $-16 + 16\sqrt{2}$

   (d) $12/3$, $16/3$, $20/3$

3. A rectangular sheet of cardboard 2 m by 1 m is used to make an open box by cutting squares of equal size from the four corners and folding up the sides. What size squares should be cut to obtain the largest possible volume?

    (a) $\dfrac{3+\sqrt{3}}{6}$ m

    (b) $\dfrac{3-\sqrt{3}}{6}$ m

    (c) $1/2$ m

    (d) $1/4$ m

4. Suppose that the number of bacteria present in a bacteria culture at time t is given by $N = 10,000e^{-t/10}$. Find the smallest number of bacteria in the culture during the time interval $0 \le t \le 50$.

    (a) 67

    (b) $10,000$

    (c) $3,679$

    (d) $73,891$

5. An object moves a distance s away from the origin according to the equation $s(t) = 4t^4 - 2t + 1$, where $0 \le t \le 10$. At what time is the object farthest from the origin?

    (a) $t = 0$

    (b) $t = 2$

    (c) $t = 10$

    (d) $t = 1/8$

6. An electrical generator produces a current, $I(t)$, starting at $t = 0$ s and running until $t = 6\pi$ s. If $I(t) = \sin(2t)$ A, then find the maximum current that is produced.

    (a) 1 A

    (b) 0 A

    (c) 2 A

    (d) $\frac{1}{2}$ A

7. A passing storm has a wind speed, $v(t)$, in mph that changes over time. If $v(t) = -t^2 + 10t + 55$ for $0 \le t \le 10$ hours, then find the highest wind speed that occurs during the storm.

    (a) 55 mph

    (b) 80 mph

    (c) 110 mph

    (d) 30 mph

8. A company has a cost of operation function given by $C(t) = 0.01t^2 - 6t + 1,000$ for $0 \le t \le 500$. Find the minimum cost of operation.

    (a) $\$1,000$

    (b) $100

    (c) $500

    (d) $0

9. Find the point on the curve $x^2 + y^2 = 4$ closest to the point $(0,3)$.

    (a) $(0,4)$

    (b) $(0,2)$

    (c) $(2,0)$

    (d) $(4,0)$

10. Answer true or false. The point on the parabola $y = x^2$ closest to $(0,9)$ is $(0,0)$.

11. For a triangle with sides 3 m, 4 m and 5 m, the smallest circle that contains the triangle has a diameter of

    (a) 3 m

    (b) 4 m

    (c) 5 m

    (d) 10 m

12. Answer true or false. If $f(t) = 3e^{4t}$ represents a growth function over the time interval $[a,b]$, then the absolute maximum must occur at $t = b$.

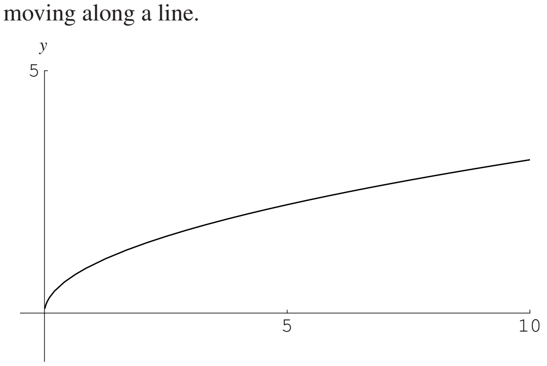
13. Answer true or false. The rectangle with the largest area that can be inscribed inside a circle is a square.

14. Answer true or false. The rectangle with the largest area that can be inscribed in a semi-circle is a square.

15. Answer true or false. An object that is thrown upward and will have a height of $s(t) = 50 + 120t - 16t^2$ for $0 \le t \le 2$. The object is highest at $t = 2$.

Section 4.6

1. The graph below represents the position function of a particle moving along a line.



Determine what is happening to the velocity of the particle at $t = 5$.

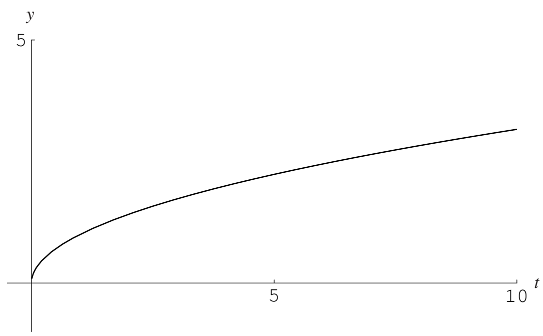
    (a) The velocity is increasing.

(b) The velocity is decreasing.

(c) The velocity is constant.

(d) Not enough information is given.

2. The graph below represents the position function of a particle moving along a line.



Which of the following statements is true about the acceleration of the particle at $t = 5$?

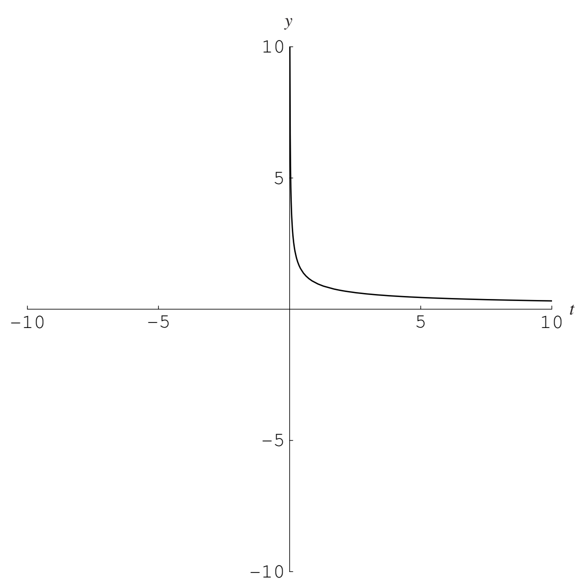
(a) Acceleration is positive.

(b) Acceleration is negative.

(c) Acceleration is zero.

(d) Not enough information is given.

3. The graph below represents the velocity function of an object.



The acceleration of the object at $t = 5$ is

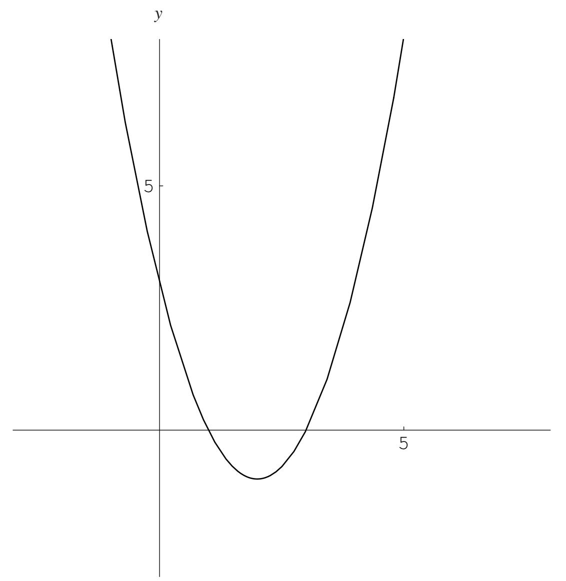
(a) positive.

(b) negative.

(c) zero.

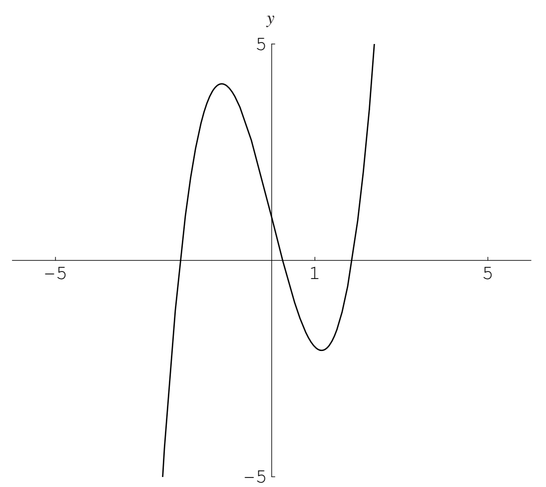
(d) not able to be determined.

4. Answer true or false.



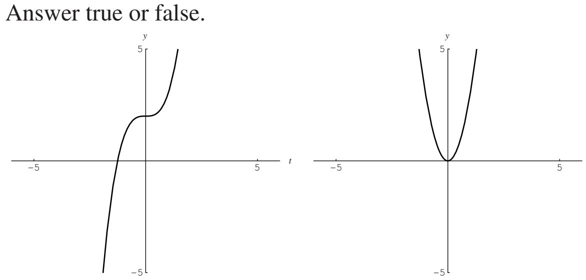
If the graph above is the position function of a particle, then the particle is moving to the right at $t = 0$.

5. Answer true or false.



For the position function graphed above, the acceleration at $t = 1$ is positive.

6. Answer true or false.



If the graph on the left is the position function, then the graph on the right represents the corresponding velocity function.

7. Let $s(t) = \cos t$ be a position function of a particle. At $t = \pi/2$ the particle's velocity is
   (a) positive
   (b) negative
   (c) zero
   (d) unknown.
8. Let $s(t) = t^3 - t$ be a position function of a particle. At $t = 0$ the particle's acceleration is
   (a) positive
   (b) negative
   (c) zero
   (d) unknown.
9. Consider the position function $s(t) = t^4 - 2t$ for $t \geq 0$. The velocity function, $v(t)$, is given by
   (a) $t^3 - 2$
   (b) $4t^3 - 2$
   (c) $12t^2$
   (d) $12t^2 - 2$
10. Consider the position function $s(t) = t^4 - 2t$ for $t \geq 0$. The acceleration function, $a(t)$, is given by
   (a) $t^3 - 2$
   (b) $4t^3 - 2$
   (c) $12t^2$
   (d) $12t^2 - 2$
11. Given the position function $s(t) = t^4 - 4t^2$ for $t \geq 0$, find t when the acceleration is zero.
   (a) $t = 12$
   (b) $t = -12$
   (c) $t = \sqrt{2/3}$
   (d) $t = -2/3$
12. Given the position function $s(t) = t^3 - 3t$ for $t \geq 0$, find t when the acceleration is zero.
   (a) $t = 1$
   (b) $t = 2$
   (c) $t = 0$
   (d) $t = -1$
13. Let $s(t) = \sqrt{3t^2 - 2}$ be a position function. Find the velocity, $v(t)$, when $t = 1$.
   (a) $v(1) = 3$
   (b) $v(1) = 6$
   (c) $v(1) = 1$
   (d) $v(1) = 0$

Section 4.7

1. Approximate $\sqrt{3}$ by applying Newton's Method to the equation $x^2 - 3 = 0$. Use $x_0 = 1$ as a starting point and use x_2 to approximate the root.
   (a) 2
   (b) 1.75
   (c) 1.73214
   (d) 1.73205
2. Approximate $\sqrt[3]{9}$ by applying Newton's Method to the equation $x^3 - 9 = 0$.
   (a) 2.08008381347
   (b) 2.08008382305
   (c) 2.08008397111
   (d) 2.08008382176
3. Use Newton's Method to approximate the solutions of $x^3 + 2x^2 - 5x - 10 = 0$.
   (a) -2.000, 2.236
   (b) -5, 0, 5
   (c) -3.1623, 0, 3.1623
   (d) -3.1623, 3.1623
4. Use Newton's Method to find the largest positive solution of $x^3 - x^2 + 2x - 4 = 0$.
   (a) 1.478
   (b) 1.000
   (c) 2.828
   (d) 3.721
5. Use Newton's Method to find the largest positive solution of $x^3 - x^2 + 3x - 3 = 0$.
   (a) 1.7325
   (b) 1.000
   (c) 1.7319
   (d) 1.7316
6. Use Newton's Method to find the largest positive solution of $x^4 + 6x^3 - x^2 - 6 = 0$.
   (a) 6.000
   (b) 1.000

(c) 1.732
(d) 1.412

7. Use Newton's Method to find the largest positive solution of $x^4+x^3-4x-4=0$.
   (a) 4.000
   (b) 1.000
   (c) 0.500
   (d) 1.587
8. Use Newton's Method to find the largest positive solution of $x^5-2x^3-14x^2+28=0$.
   (a) 3.742
   (b) 2.410
   (c) 1.414
   (d) 1.260
9. Use Newton's Method to find the largest positive solution of $x^5+2x^3-2x^2-4=0$.
   (a) 1.260
   (b) 1.414
   (c) 1.587
   (d) 2.000
10. Use Newton's Method to find the largest positive solution of $x^4-13x^2+30=0$.
   (a) 3.162
   (b) 2.340
   (c) 5.477
   (d) 1.732
11. Use Newton's Method to find the largest positive solution of $x^5+x^4+x^3-5x^2-5x-5=0$.
   (a) 2.236
   (b) 1.380
   (c) 1.710
   (d) 1.621
12. Use Newton's Method to find the x-coordinate of the intersection of $y=2x^3-2x^2$ and $y=-x^5+4$.
   (a) 3.742
   (b) 2.410
   (c) 1.414
   (d) 1.260
13. Use Newton's Method to find the greatest x-coordinate of the intersection of $y=x^4-7x^2$ and $y=6x^2-30$.
   (a) 3.162
   (b) 2.340
   (c) 5.477
   (d) 1.732

Section 4.8

1. Answer true or false. $f(x)=\dfrac{1}{x}$ on $[-1,1]$ satisfies the hypotheses of Rolle's Theorem.
2. Find the value c such that the conclusion of Rolle's Theorem are satisfied for $f(x)=x^2-4$ on $[-2,2]$.
   (a) 0
   (b) -1
   (c) 1
   (d) $1/2$
3. Answer true or false. The Mean-Value Theorem is used to find the average of a function.
4. Answer true or false. The Mean-Value Theorem can be used on $f(x)=|x|$ on $[-2,1]$.
5. Answer true or false. The Mean-Value Theorem guarantees there is at least one such c on $[0,1]$ such that $f'(c)=1$ when $f(x)=\sqrt{x}$.
6. If $f(x)=\sqrt[3]{x}$ on $[0,1]$, find the value c that satisfies the Mean-Value Theorem.
   (a) 1
   (b) $1/3$
   (c) $\left(\dfrac{1}{3}\right)^{3/2}$
   (d) $1/9$
7. Answer true or false. The hypotheses of the Mean-Value Theorem are satisfied for $f(x)=\sqrt[3]{|x|}$ on $[-1,1]$.
8. Answer true or false. The hypotheses of the Mean-Value Theorem are satisfied for $f(x)=\sin x$ on $[0,4\pi]$.
9. Answer true or false. The hypotheses of the Mean-Value Theorem are satisfied for $f(x)=\dfrac{1}{\sin x}$ on $[0,4\pi]$.
10. Find the x-value for which $f(x)=x^2+3$ on $[1,3]$ satisfies the Mean-Value Theorem.
   (a) $x=2$
   (b) $x=9/4$
   (c) $x=7/3$
   (d) $x=11/4$
11. Find the x-value for which $f(x)=x^3$ on $[2,3]$ satisfies the Mean-Value Theorem.

(a) 2.517

(b) 2.500

(c) 2.250

(d) 2.125

12. Answer true or false. A graphing utility can be used with Rolle's Theorem to show that the function $f(x) = (x-2)^2$ has a point where $f'(x) = 0$.
13. Answer true or false. According to Rolle's Theorem, if a function does not cross the x-axis, then its derivative cannot be zero anywhere.
14. Find the value c that satisfies Rolle's Theorem for $f(x) = \sin x$ on $[0, \pi]$.

    (a) $\pi/4$

    (b) $\pi/2$

    (c) $3\pi/4$

    (d) $\pi/3$
15. Find the value c that satisfies the Mean-Value Theorem for $f(x) = x^3 + 3x$ on $[0, 1]$.

    (a) $\frac{\sqrt{3}}{3}$

    (b) $\frac{\sqrt{3}}{2}$

    (c) $\frac{\sqrt{2}}{2}$

    (d) $\frac{\sqrt{2}}{3}$

Chapter 4 Test

1. The largest interval on which $f(x) = x^2 - 4x + 7$ is increasing is

    (a) $[0, \infty)$

    (b) $(-\infty, 0]$

    (c) $[2, \infty)$

    (d) $(-\infty, 2]$
2. Answer true or false. The function $f(x) = \sqrt{x-2}$ is concave down on its entire domain, except at $x = 2$.
3. The function $f(x) = x^3 - 8$ is concave down if

    (a) $x < -2$

    (b) $x > 2$

    (c) $x < 0$

    (d) $x > 0$
4. Answer true or false. The function $f(x) = x^4 - 2x + 3$ has a point of inflection.
5. The function $f(x) = |x^2 - 4|$ is concave up on

    (a) $(-\infty, -2) \cup (2, \infty)$

    (b) $(-\infty, -4) \cup (4, \infty)$

    (c) $(-2, 2)$

    (d) $(-4, 4)$
6. The largest open interval on which $f(x) = e^{2x^6}$ is concave up is

    (a) $(-\infty, 0)$

    (b) $(0, \infty)$

    (c) $(-\infty, \infty)$

    (d) $(-\infty, e)$
7. Determine where $f(x) = \sin x$ is increasing on the interval $[0, 2\pi]$.

    (a) $0 \le x \le \pi$

    (b) $\pi \le x \le 2\pi$

    (c) $\pi/2 \le x \le 3\pi/2$

    (d) $0 \le x \le \pi/2$ or $3\pi/2 \le x \le 2\pi$
8. Answer true or false. The function $f(x) = x^4 - 2x^2 + 10$ has at least one point of inflection.
9. The function $f(x) = -x^4 + 6x^3 - 12x^2$ is concave down on

    (a) $(-\infty, \infty)$

    (b) $(-\infty, 1) \cup (2, \infty)$

    (c) $(1, 2)$

    (d) nowhere.
10. Answer true or false. If $f''(-1) = 4$ and $f''(1) = 4$, and if f is continuous on $[-1, 1]$, then there is a point of inflection on $(-1, 1)$.
11. Determine the x-coordinate of each stationary point of $f(x) = 4x^4 - 16$.

    (a) $x = -1$

    (b) $x = 0$

    (c) $x = 16$

    (d) $x = 1$
12. Answer true or false. The function $f(x) = x^{4/9}$ has a critical point at $x = 0$.
13. The curve $y = x^2 - 8x + 9$ has

    (a) a relative maximum at $x = 4$

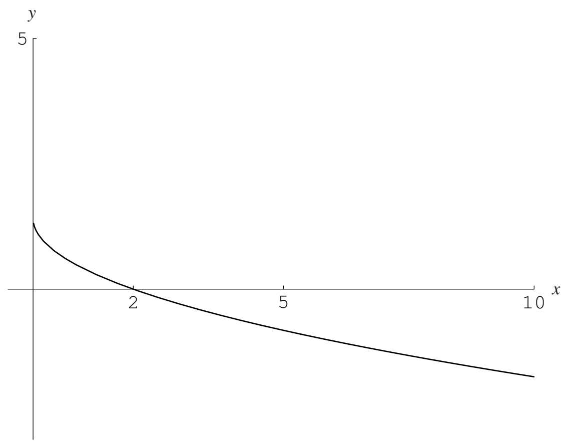
    (b) a relative minimum at $x = 4$

    (c) a relative maximum at $x = -4$

    (d) a relative minimum at $x = -4$

14. The curve $y = -e^{7x}$ has
    (a) a relative maximum at $x = 0$
    (b) a relative minimum at $x = 0$
    (c) a relative maximum at $x = 7$
    (d) no relative extrema
15. The function $f(x) = 16x^2$ has
    (a) no relative maxima; a relative minimum at $x = 4$
    (b) a relative maximum at $x = 4$; no relative minima
    (c) a relative maximum at $x = 0$; relative minima at $x = -4$ and $x = 4$
    (d) no relative maxima; a relative minima at $x = 0$
16. Answer true or false. The function $f(x) = -e^{2x}\ln(2x)$ has a relative minimum on $(0, \infty)$.
17. The rational function $f(x) = \dfrac{4x+20}{x^2-25}$ has
    (a) a horizontal asymptote at $y = 0$.
    (b) horizontal asymptotes at $y = 5$ and $y = -5$.
    (c) a horizontal asymptote at $y = 4$.
    (d) a horizontal asymptote at $y = -4/5$.
18. Answer true or false. The function $f(x) = \dfrac{3}{x-5}$ has a vertical asymptote.
19. Answer true or false. $\lim\limits_{x\to 0^+} \left(\sqrt[3]{x}\ln x\right) = 0$.
20. A weekly profit function for a company that manufactures umbrellas is $P(x) = -0.01x^2 + 3x - 2,000$, where x is the number of umbrellas that are made and sold each week. How many individual umbrellas must the company make and sell each week to maximize their profit?
    (a) 300 umbrellas
    (b) 150 umbrellas
    (c) 600 umbrellas
    (d) 60 umbrellas
21. The function $f(x) = 6x^2 - 2$ has an absolute minimum value on $[-3, 3]$ of
    (a) 2
    (b) -2
    (c) 52
    (d) -52
22. The function $f(x) = x^3 + 3$ has an absolute maximum value on $[-2, 2]$ of
    (a) 0
    (b) 6
    (c) 11
    (d) 8
23. The function $f(x) = 3\sin(x+2)$ has an absolute minimum value of
    (a) -2
    (b) -3
    (c) $-1/3$
    (d) $-2/3$
24. The function $f(x) = x^{-5}$ has an absolute maximum value on $[1, 3]$ of
    (a) 1
    (b) $1/243$
    (c) 243
    (d) There is no absolute maximum on $[1, 3]$.
25. Answer true or false. The function $f(x) = x^{-7}$ has an absolute maximum value of 1 on $[-1, 1]$.
26. Express the number 40 as the sum of two nonnegative numbers, a and b (with $a < b$), whose product is as large as possible.
    (a) $a = 5, b = 35$
    (b) $a = 10, b = 30$
    (c) $a = 20, b = 20$
    (d) $a = 1, b = 39$
27. The position of an object relative to the origin is given by $s(t) = t^4 - 2$ for $0 \leq t \leq 10$. At what time is the object farthest from the origin?
    (a) $t = 0$
    (b) $t = 2$
    (c) $t = 8$
    (d) $t = 10$
28. Find the point on the curve $x^2 + y^2 = 16$ closest to $(0, 5)$.
    (a) $(0, 4)$
    (b) $(4, 0)$
    (c) $(-4, 0)$
    (d) $(0, -4)$
29. Answer true or false. A growth function $f(x) = 4e^{0.02t}$ on the interval $[0, 10]$ has an absolute maximum at $t = 10$.

30. The graph below represents the velocity function of a particle moving along a line.



Which of the following statements is true about the motion of the particle?

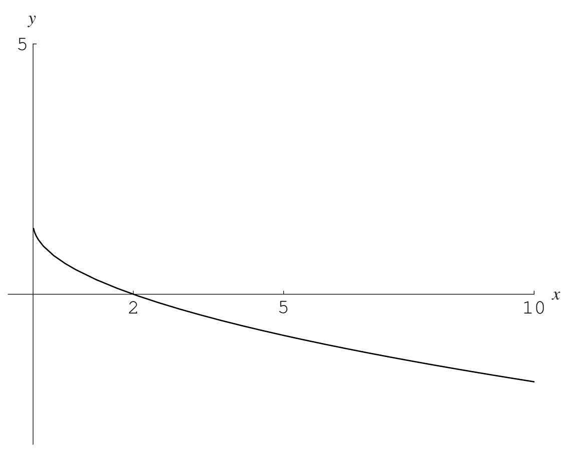
(a) The particle is at the origin at $t = 2$.

(b) The particle is moving to the right for $0 \leq t < 2$.

(c) The particle is always moving to the left.

(d) The particle is slowing down when $t > 2$.

31. The graph below represents the velocity function of a particle along a line.



Which of the following statements is true about the acceleration of the particle?

(a) The acceleration is positive for $t \geq 0$.

(b) The acceleration is negative for $t \geq 0$.

(c) The acceleration is positive on $[0,2)$ and negative on $(2,\infty)$

(d) Not enough information is given.

32. Let $s(t) = t^4 - 2$ be the position function of a particle. The particle's acceleration for $t > 0$ is

(a) positive.

(b) negative.

(c) zero.

(d) Not enough information is given.

33. Let $s(t) = 4 - t^2$ be the position function of a particle. The particle's acceleration for $t > 0$ is

(a) positive.

(b) negative.

(c) zero.

(d) Not enough information is given.

34. Let the position function for a particle be $s(t) = 4t^2 - 8$. Find t when the acceleration of the particle is zero.

(a) $t = 0$

(b) $t = 8$

(c) $t = 2$

(d) Acceleration is never zero.

35. Approximate $\sqrt{11}$ using Newton's Method.

(a) 3.31662479036

(b) 3.31662478727

(c) 3.31662479002

(d) 3.31662478841

36. Use Newton's Method to approximate the greatest positive solution of $x^3 + 4x^2 - 5x - 20 = 0$.

(a) 4.000

(b) 2.236

(c) 5.292

(d) 3.037

37. Answer true or false. The hypotheses of Rolle's Theorem are satisfied for $f(x) = x^{-6} - 1$ on $[-1,1]$.

38. Answer true or false. Given $f(x) = x^2 - 16$ on $[-4,4]$, the value c that satisfies Rolle's Theorem is $c = 0$.

39. Answer true or false. Let $f(x) = x^5$ on $[-1,1]$. The value c that satisfies Rolle's Theorem is $c = 0$.

Chapter 4: Answers to Sample Tests

Section 4.1

1. false	2. false	3. a	4. c	5. c	6. a	7. c	8. d
9. d	10. c	11. true	12. false	13. d	14. false	15. d	

Section 4.2

1. b	2. b	3. true	4. false	5. b	6. a	7. b	8. false
9. d	10. c	11. b	12. c	13. false	14. true	15. false	

Section 4.3

1. false	2. a	3. a	4. a	5. false	6. c	7. b	8. a
9. c	10. b	11. false	12. true	13. a	14. true	15. true	

Section 4.4

1. a	2. a	3. false	4. false	5. a	6. d	7. d	8. b
9. a	10. false	11. a	12. false	13. false	14. b	15. a	

Section 4.5

1. c	2. c	3. b	4. a	5. c	6. a	7. b	8. b
9. b	10. false	11. c	12. true	13. true	14. false	15. false	

Section 4.6

1. b	2. b	3. b	4. false	5. true	6. true	7. b	8. c
9. b	10. c	11. c	12. c	13. a			

Section 4.7

1. b	2. b	3. a	4. a	5. b	6. b	7. d	8. b
9. a	10. a	11. c	12. d	13. a			

Section 4.8

1. false	2. a	3. false	4. false	5. true	6. c	7. false	8. true
9. false	10. a	11. a	12. false	13. false	14. b	15. a	

Chapter 4 Test

1. c	2. true	3. c	4. false	5. a	6. c	7. d	8. false
9. b	10. false	11. b	12. true	13. b	14. d	15. d	16. false
17. a	18. true	19. true	20. b	21. b	22. c	23. b	24. a
25. false	26. c	27. d	28. a	29. true	30. b	31. b	32. a
33. b	34. d	35. a	36. b	37. false	38. true	39. false	