Chapter 6: Applications of the Definite Integral in Geometry, Science and Engineering

Summary: This chapter focuses upon using the methods of evaluating definite integrals and applying them in various problems. The first problem considered is that of finding the area between two curves. This extends the idea of finding the area underneath a curve or the total area between a function and an interval. The next application is to find the volumes of various objects or solids. Three basic methods are introduced: the method of slicing, the method of washers (or disks) and the method of shells. The latter two methods may only be applied in the case of a volume of revolution when a curve is revolved around a particular axis. After that, finding the length of a curve or the arc length is discussed. Next the surface area of a solid of revolution is investigated and then the average value of a function. Towards the end of the chapter, two physical applications are discussed: work and force from fluid pressure. At the very end of the chapter, hyperbolic functions such as $\sinh x$ and $\cosh x$ are defined and studied.

OBJECTIVES: After reading and working through this chapter you should be able to do the following:

1. Use definite integrals to find the area between two curves (§6.1).
2. Use the method of slicing to find the volume of a solid (§6.2).
3. Use the method of disks/washers to find the volume of a solid of revolution (§6.2).
4. Use the method of cylindrical shells to find the volume of a solid of revolution (§6.3).
5. Find the arc length of a plane curve (§6.4).
6. Find the surface area of a solid of revolution (§6.5).
7. Calculate work done by constant and variable forces over a distance (§6.6).

8. Find the center of gravity and centroid of a two dimensional region or thin lamina (§6.7).
9. Calculate the force due to fluid pressure on objects submerged in liquid (§6.8).
10. Learn the definitions and properties of the hyperbolic functions (§6.9).

6.1 Area Between Two Curves

PURPOSE: To use definite integrals to calculate the area between two curves.

area problem
(see §5.1, 5.4 – 5.6)

Now that **area** can be described using definite integrals (from the previous chapter), the next step is to use definite integrals to describe the area between two curves. The process is rather straightforward. In the case of finding the area under a single function, this can be thought of as finding the areas of many infinitesmal rectangles between the function and the axis. Now, these rectangles will be between the two curves. Then the general area function can be stated as

$$\int_a^b (f_1(x) - f_2(x))\ dx$$

where $f_1(x) > f_2(x)$.

IDEA: Finding the area between two curves is like adding together the areas of many small rectangles that lie between a top function and a bottom function.

However, if the two functions trade places then their position in the formula needs to change (i.e., if $f_2 > f_1$ then use $\int_a^b (f_2(x) - f_1(x))\ dx$). The function on the bottom is always being subtracted from the one on the top. This may require breaking the integral into several smaller integrals with different limits of integration and slightly different integrands.

IDEA: Limits of integration may depend upon where the two functions intersect. Limits are chosen so that the "bottom" function is always being subtracted from the "top."

Limits of integration will either be given or they must be found. It always must be determined if the functions intersect. The points of intersection will help to determine the limits of integration necessary on the integrals as well as helping to determine which function is the top and which is the bottom.

Another way of thinking of the area between two functions is to define a new function: $h(x) = f_1(x) - f_2(x)$. For example, $h(x)$ might represent the height of one of the small rectangles between the two functions. Then the area between the two functions can be written as

$$\int_a^b |h(x)|\ dx$$

The integrand, $|h(x)|\ dx$ represents the area of one of the infinitesmal rectangles and the integral is then summing together all of the rectangle areas. The absolute value ensures that the function on the bottom is being subtracted from the function on the top so that each rectangle has a positive height.

It is also a simple matter to change these formulas so that they are written with respect to y instead of x. The functions used as integrands need to be written as functions of y instead of x and the limits of integration need to be changed to y values instead of x values. Otherwise, the process is the same even though visually it may appear to be different.

vertical rectangles between $f_1(x)$ and $f_2(x) \rightarrow\ dx$ width

horizontal rectangles between $g_1(y)$ and $g_2(y) \rightarrow\ dy$ width

Checklist of Key Ideas:

- ☐ area between two curves
- ☐ calculating area using definite integrals; finding the limits of integration
- ☐ area formulas with respect to x and/or y

6.2 Volumes by Slicing; Disks and Washers

PURPOSE: To calculate the volumes of solids oriented along a particular axis.

Finding the volume of different solids is very closely related to the process of finding the area under a curve or the area between two curves. In each case, the idea is to find a way to slice the solid into smaller shapes that can be described by a definite integral. The slices that are used will all be perpendicular to some common axis.

IDEA: Each object studied here will be divided up into slices so that each slice is perpendicular to either the x or y-axis.

The most general is the method of **volume by slicing**. In this case, either the cross-section of each slice is given or can be found geometrically. Then the area of the cross-section, $A(x)$, is the integrand.

volume by slicing

$$V = \int_a^b A(x)\ dx$$

The volume of each slice can be thought of as $A(x)\ dx$ and then the integral is simply adding together all of the slices.

The **method of disks** and the **method of washers** are more specific applications of the method of slicing. If the solid is the result of revolving a curve about an axis then the cross-sections will all be circular in shape. Then $A(x) = \pi\ [f_1(x)]^2$ (in the method of disks) or $A(x) = \pi\big([f_1(x)]^2 - [f_2(x)]^2\big)$ (in the method of washers). Here the function values are the radius of each of the circles (i.e., the distance to the x-axis).

disk and washers

IDEA: The method of disks is a specific case of the method of washers where the inner function is zero, i.e., $f_2(x) = 0$.

The reader may notice that the method of disks is just a specific case of the method of washers where the $f_2(x) = 0$. Also, both the disk and washer methods are directly related to the area problem from the previous section. If $h_1(x) = \pi\,[f_1(x)]^2$ and $h_2(x) = \pi\,[f_2(x)]^2$ then the method of washers is just the total area between the functions h_1 and h_2.

revolving about y-axis

All of the above descriptions are given in terms of revolving a function about the x-axis. If a function is **revolving about the y-axis** then the integrals need to be written with respect to y and the functions need to be functions of y. Also the limits need to be written as values of y.

Here is a general summary of the method of washers (and disks if f_2 or g_2 is zero):

Revolving around x-axis:

$$V = \int_{x=a}^{x=b} \pi\left([f_1(x)]^2 - [f_2(x)]^2\right)\,dx \qquad \text{if } f_1 \geq f_2 \geq 0$$

Revolving around the line $y = k$ (parallel to the x-axis):

$$V = \int_{x=a}^{x=b} \pi\left([f_1(x) - k]^2 - [f_2(x) - k]^2\right)\,dx \qquad \text{if } f_1 \geq f_2 \geq k$$

$$V = \int_{x=a}^{x=b} \pi\left([k - f_2(x)]^2 - [k - f_1(x)]^2\right)\,dx \qquad \text{if } k \geq f_1 \geq f_2$$

Revolving around y-axis:

$$V = \int_{y=c}^{y=d} \pi\left([g_1(y)]^2 - [g_2(y)]^2\right)\,dy \qquad \text{if } g_1 \geq g_2 \geq 0$$

Revolving around the line$x = k$ (parallel to y-axis):

$$V = \int_{y=c}^{y=d} \pi\left([g_1(y) - k]^2 - [g_2(y) - k]^2\right)\,dy \qquad \text{if } g_1 \geq g_2 \geq k$$

$$V = \int_{y=c}^{y=d} \pi\left([k - g_2(y)]^2 - [k - g_1(y)]^2\right)\,dy \qquad \text{if } k \geq g_1 \geq g_2$$

Checklist of Key Ideas:

- ☐ volume by slicing; general volume formulas (with respect to x and/or y)
- ☐ volume of a right cylinder; volume of a disk
- ☐ axis perpendicular to disks
- ☐ solids of revolution
- ☐ method of disks; method of washers

6.3 Volumes by Cylindrical Shells

PURPOSE: To use integrals to compute the volume of a solid using cylindrical shells.

The method of disks and shells are designed specifically for the situation where a solid is obtained by a revolution of a curve about a particular axis (usually the x- or y-axis or some line that is parallel to one of them). The **method of shells** is based upon the volume of a right circular cylinder. The volume of one shell is given by

method of shells

$$\text{shell volume} = 2\pi(x-a)f(x)\,dx$$

where the shell has a center along the line at $x = a$. Then the volume formula will be given by

$$V = \int_{x=a}^{x=b} 2\pi(x-a)f(x)\,dx$$

Notice that in this example the solid is revolved about a line $x = a$ which is parallel to the y-axis.

IDEA: Washers and shells require integration with respect to the opposite variable. In some cases this may make one method easier than the other.

washers vs. shells

Using the **method of washers** in this case would have used integration with respect to y. Because of this difference, it may be easier to apply the **method of shells** than the method of washers in some cases. Another difference between the method of washers (or disks) and shells is that the method of washers requires that the function be squared while the method of shells requires that the function be multiplied by a distance (usually x, for example).

Checklist of Key Ideas:

- ☐ volume of a cylindrical shell; difference between shells and disks
- ☐ method of cylindrical shells
- ☐ volume formula about the x- or y-axis
- ☐ units of the integral

6.4 Length of a Plane Curve

PURPOSE: To use integrals to find the length of a piece of a curve.

There is only one idea that is introduced in this section: how to find the length of a piece of a curve. The formula that is used is straightforward and requires that some integrals may require numerical methods (see §5.4, 7.7) or CAS (see §7.6)

the derivative of the function be obtained. Often the integral that results cannot be evaluated without special means such as a **numerical method** or **CAS**. The fact that the derivative is required indicates when this formula may be applied. The function in question must have a derivative over the entire length of the curve or else the formula may return faulty values.

IDEA: To calculate arc length requires that the appropriate derivatives of the curve can be calculated.

Three arc length formulas

1. with respect to x, $y = f(x)$
2. with respect to y, $x = g(y)$
3. parametrically (see also §10.1)

Three formulas are given for finding the length of a curve. One is **with respect to** x and one is **with respect to** y. Arc length can also be calculated **parametrically**. This will also be discussed later in Section 10.1.

$$\int_a^b \sqrt{1+(dy/dx)^2}\, dx \qquad \text{with respect to } x$$

$$\int_c^d \sqrt{1+(dx/dy)^2}\, dy \qquad \text{with respect to } x$$

$$\int_a^b \sqrt{(dx/dt)^2+(dy/dt)^2}\, dt \qquad \text{parametrically}$$

In some cases, writing the length with a different integral can be the difference between getting an exact answer and having to evaluate the definite integral numerically.

Checklist of Key Ideas:

- ☐ arc length problem
- ☐ smooth curves/functions
- ☐ formula for arc length (with respect to x and/or y)
- ☐ arc length formula for curves defined parametrically
- ☐ units of the integral

6.5 Area of a Surface of Revolution

PURPOSE: To use integrals to compute the area of a surface of revolution.

After discussing solids of revolution and the length of an arc, the area of a surface of revolution can be discussed. There is only one formula that is introduced in this section: finding the surface area of a solid of revolution.

frustum
slant height

The area formula is based upon the surface area of a **frustum** of **slant height** l: area $= \pi(r_1+r_2)l$. The slant height of the frustum can be found using the incremental arc length of $l = \sqrt{1+[f'(x)]^2}\, dx$.

IDEA: The slant height of the frustum is found using the arc length ideas from the previous section. This means that the appropriate derivative needs to exist.

The radius of the top and bottom of the frustum are approximately the same and so the area becomes

$$\text{area} = \pi(2f(x))\sqrt{1+[f'(x)]^2}\,dx$$

Then summing these areas, the surface area can be written as

$$A = \int_{x=a}^{x=b} 2\pi f(x)\sqrt{1+[f'(x)]^2}\,dx$$

Often this formula cannot be evaluated directly or will need u-substitution.

numerical methods (see §5.4, 7.7)
CAS (see §7.6)

Checklist of Key Ideas:

- ☐ surface area problem
- ☐ surface of revolution
- ☐ formula for finding area of a surface of revolution

6.6 Work

PURPOSE: To use integrals to compute the work done by a force.

The **units of work** are energy and work is done by applying a force over a distance. The units of energy (and work) can be found by multiplying together the units of force and the units of distance.

units of work

IDEA: Three types of force:

1. constant force
2. variable force
3. many cases of constant force

In this section, **three types of work** are considered: with a constant force, with a variable force, and adding together many cases where the force is constant. When the **force is constant**, then the work done is simply the product of the force, F, and the distance, d, over which the force is applied.

constant force

$$W = F \cdot d$$

When the **force is variable**, then the work done is the integral of the force, $F(x)$:

variable force

$$\text{work} = \int_{x=a}^{x=b} F(x)\ dx$$

Notice that the units of $F(x)\ dx$ will be force times length which should equal energy.

IDEA: Two special types of work:

1. work done by springs
2. work of pumping a liquid

work and springs

Two special types of work are considered in this section: work done on a spring and work done pumping liquid. For **springs**, the force that is applied is a variable force that is proportional to the amount that the spring is compressed or stretched, $F(x) = kx$, where k is the spring constant.

pumping liquids

lifting=applying a force

For **pumping liquids**, the total work can be thought of as many separate instances of work being done with a constant force that are then added together. This is very similar to the volume by slicing method introduced earlier in this chapter (see §6.2). A volume of liquid can be divided into thin layers of liquid that have a cross-sectional area of $A(x)$ and a thickness of dx so that each layer has a volume of $A(x)\ dx$. Then multiplying the volume of each layer by its weight density will give the weight of the layer. The process of pumping is then to **lift each layer of liquid a certain distance**.

IDEA: Thinking of the liquid as many different layers is similar to finding the volume of an object by slicing. In this problem, the slices or layers are also being moved some distance. So two key ideas are

1. the size of each "layer" of liquid
2. how far the "layer" is being moved

For both pumping and spring problems, care should be taken when choosing what the x-axis represents. In spring problems, $x = 0$ represents when the spring is at its natural length. Then $x = 1$ would represent when the spring is stretched beyond, or compressed from, its natural length by one unit.

IDEA: Picking an x-axis can usually be done in more than one way. The important thing is to be consistent throughout the problem.

In pumping problems, one convention (as used by this book) is to have the positive x direction to be in the downward direction. Where $x = 0$ is located is arbitrary and is often left up to the reader. Usually $x = 0$ is chosen to be in a position that makes other pieces of the integrand to be relatively simple expressions. However the pumping is done, the lower limit of integration should be less than the upper limit of integration and the distance that each layer of liquid is pumped should be positive. The result should be a positive amount of work done. Also, the limits of integration are chosen based upon where the liquid is located on the x-axis, not on where the liquid is being pumped to.

IDEA: Be careful with the distance and the limits of integration. The overall work in lifting liquid should be a positive amount of energy.

Lastly, **work is equivalent to the change in kinetic energy** of an object. This is shown by the equation

work is change in kinetic energy

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

where v_f is the final velocity of an object and v_i is the initial velocity of an object. When calculating work based upon change in velocity, no integrals have to be evaluated. The work may be calculated by evaluating the above expression directly.

Checklist of Key Ideas:

- ☐ work-energy relationship; kinetic energy
- ☐ work of a constant force over a distance
- ☐ work of a variable force over a distance
- ☐ springs and spring constants
- ☐ pumping liquid

6.7 Moments, Centers of Gravity, and Centroids

PURPOSE: To find the center of gravity of an inhomogeneous lamina or the centroid of a homogeneous lamina.

This section discusses how to find the center of gravity of a thin, two-dimensional region called a **lamina**. Since this region is two-dimensional we will describe the region using equations in the xy-plane. The lamina may have some non-constant density function, $\delta(x)$, in which case the lamina is said to be **inhomogeneous**. Otherwise the lamina is called **homogeneous**. The center of gravity is the point in the region upon which the region can be balanced. In other words, the force due to gravity on the region can be seen as a single force that acts on the region through its center of gravity. When a lamina is homogeneous, the center of gravity is called the centroid.

lamina
inhomogeneous
homogeneous

center of gravity
centroid

IDEA: The integrals for M, M_x and M_y are required to calculate the center of gravity or centroid.

$$\bar{x} = \frac{M_y}{M} \text{ and } \bar{y} = \frac{M_x}{M}$$

Three integral formulas are necessary to calculate the center of gravity of a lamina: the **moment about the x-axis** (M_x), the **moment about the y-axis** (M_y), and the

M, mass
M_x, moment about x-axis
M_y, moment about y-axis

mass (M) of the region. The integrals required to calculate these are very similar to integrals used in other applications in this chapter.

IDEA: Setting up the integral for mass, M, of a laminar region R is very similar to setting up the integral for finding the area of the region.

For example, finding the mass of a region is very similar to finding the area of a region. Finding the moments about the x-axis and y-axis are very similar to the integrals that arise when finding the volume of a solid of revolution.

IDEA: Setting up the integrals for M_x and M_y are very similar to the integrals that are used to find volumes for solids of revolution about the x-axis and y-axis respectively.

Suppose that a region R is bounded by the curves $y = f(x)$ at the bottom, $y = g(x)$ at the top, and $x = a$ and $x = b$ at the left and right. When integrating with respect to x, the area of this region is given by the following integral $\int_a^b (g(x) - f(x))\, dx$. Calculating the mass simply inserts the density, δ, into this integral:

$$M = \int_a^b \delta \cdot (g(x) - f(x))\, dx.$$

Also, the moment about the x-axis,

$$M_x = \frac{1}{2} \int_a^b \delta \cdot (g(x)^2 - f(x)^2)\, dx,$$

looks remarkably similar to integration the washer method when it is used to find the volume of the solid of revolution about the x-axis. Likewise, the moment about the y-axis,

$$M_y = \int_a^b x \cdot \delta \cdot (g(x) - f(x))\, dx$$

looks much like the volume when revolving about the y-axis if the shell method is used. If integration with respect to y is used, then M_x will look like the shell method being used around the x-axis and M_y will resemble the washer method being used around the y-axis.

centroids depends only upon the shape of lamina

The centroid is found in essentially the same way as the center of gravity. There will be no real difference in the calculation other than the density of the region will be constant. This will cause the density to divide out of the calculation of $\bar{x}$ and $\bar{y}$. Since the centroid is then independent of the density, the location of the centroid is considered to be a geometric property of the region. That is, only the shape of the region has an impact upon the location of the centroid.

IDEA: The Theorem of Pappus allows the centroid of a region to be used as a convenient tool for finding the volume of a solid of revolution.

volume $=$ (circumference travelled by centroid) $\times$ (area of region)

Theorem of Pappus
solids of revolution

A useful application of the centroid is found in calculating the volume of a solid of revolution. **The Theorem of Pappus** essentially says that the volume is equal to the area of the region times the distance that the centroid travels when the region is revolved. To find the distance that the centroid travels, imagine its path as a circle. The distance it is revolved will be equal to the circumference of the circle. The radius of this circle is the straight distance from the centroid to the line being revolved around. Any line can be considered, not just vertical and horizontal lines.

Checklist of Key Ideas:

- ☐ inhomogeneous lamina
- ☐ homogeneous lamina
- ☐ moments about the x-axis and y-axis
- ☐ center of gravity
- ☐ centroid
- ☐ mass of a lamina
- ☐ Theorem of Pappus

6.8 Fluid Pressure and Force

PURPOSE: To use integrals to calculate the total force that is applied by a fluid over an area.

The idea presented in this section is force applied over an area and finding the total force that is applied by a fluid on a submerged object or surface. As in the previous section, the **units** that are being used can help to straighten out how to write the integrals involved.

units

The basic formula being used is the relationship between force, F, pressure, P, and area A.

$$F = PA \quad \text{or} \quad P = \frac{F}{A}$$

Thus the units of pressure, P, are force per area. In the SI system this is typically given as Pascals or Pa. In the BE system, this is usually given as pounds per square inch or PSI.

The main problem considered here is how to calculate the force on a submerged surface whether it is horizontal, vertical or inclined at some angle (see the exercises). The direction of the force on the surface is known to be perpendicular to the surface in question.

If it is a **horizontal surface** then the force is just equal to the pressure times the area. The pressure is shown to be the weight density, ρ, times the distance, h, at which the surface is submerged. Then the force is given as follows.

horizontal surface

$$F = \rho h A$$

vertical surface

If the **surface is vertical** then the force is calculated using an integral. The x-axis is taken to be positive in the downward direction. $x = 0$ does not necessarily need to be at the surface of the liquid. The submerged surface is divided into horizontal strips which have a thickness of dx and a width given by $w(x)$ giving each horizontal strip an area of $w(x)\ dx$. Each strip has a depth of $h(x)$ below the surface of the liquid. Then each strip will experience a force of $\rho\ h(x)w(x)\ dx$. This is integrated with limits of integration based upon where the submerged surface is on the x-axis.

$$\int_{x=a}^{x=b} \rho h(x)w(x)\ dx$$

Here $x = a$ is at the top of the surface and $x = b$ is at the bottom of the submerged surface. As a check, the units of $\rho h(x)w(x)\ dx$ should be units of force.

Checklist of Key Ideas:

- ☐ definition of pressure
- ☐ fluid density; mass/weight density
- ☐ fluid pressure on horizontal or vertical surface
- ☐ formula for finding fluid force
- ☐ units of the integral

6.9 Hyperbolic Functions and Hanging Cables

PURPOSE: To define the various hyperbolic trigonometric functions and to discuss their properties.

$\sinh u = \dfrac{e^u - e^{-u}}{2}$

$\cosh u = \dfrac{e^u + e^{-u}}{2}$

$\tanh u = \dfrac{\sinh u}{\cosh u}$

Hyperbolic functions are introduced and defined in this section. These functions are not really an application of integrals but rather are a new group of functions which are defined as combinations of exponential functions. Everything that is shown in this section can be found by remembering the definitions of each function. For example, since $\sinh x = \dfrac{e^x - e^{-x}}{2}$ then its derivatives and antiderivatives can be found by simply knowing this information for e^x and e^{-x}. Once this type of information is known for $\sinh x$ and $\cosh x$, then similar results can be obtained for $\tanh x = \dfrac{\sinh x}{\cosh x}$.

Other important features can be remembered by thinking of $\sinh x$ and $\cosh x$ in pieces:

$$\begin{aligned} \sinh x &= \frac{e^x}{2} - \frac{e^{-x}}{2} \\ \cosh x &= \frac{e^x}{2} + \frac{e^{-x}}{2} \end{aligned}$$

Recall that $e^x \to 0$ as $x \to -\infty$ and $e^{-x} \to 0$ as $x \to \infty$. Then $\sinh x$ and $\cosh x$ become asymptotically close to $e^x/2$ as $x \to \infty$. On the other hand, $\sinh x$ becomes asymptotically close to $-e^{-x}/2$ as $x \to -\infty$ and $\cosh x$ becomes asymptotically close to $e^{-x}/2$ as $x \to -\infty$. Similar properties of $\tanh x$ and the other hyperbolic functions by either defining them in terms of e^x and e^{-x} or in terms of $\sinh x$ and $\cosh x$.

IDEA: All the definitions can be written in terms of $\sinh x$ and $\cosh x$.

For example,

$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \operatorname{sech} x = \frac{1}{\cosh x}$$

$$\coth x = \frac{\cosh x}{\sinh x} \qquad \operatorname{csch} x = \frac{1}{\sinh x}$$

IDEA: Make notecards with the various definitions and properties of the hyperbolic functions to help remember them.

If the reader is not familiar with the material in this section, then it is suggested that the reader make a few notecards with any new information and have them handy as they attempt the exercises at the end of the section.

Checklist of Key Ideas:

- ☐ e^x as a sum of even and odd functions
- ☐ hyperbolic functions
- ☐ curvilinear asymptotes
- ☐ catenary; hanging cables
- ☐ hyperbolic identities
- ☐ hyperbolic inverses and derivatives
- ☐ logarithmic forms of inverse hyperbolic functions

Chapter 6 Sample Tests

Section 6.1

1. Find the area of the region enclosed by the curves $y = x^2$ and $y = -x$ by integrating with respect to x.
   (a) $1/6$
   (b) 1
   (c) $1/4$
   (d) $1/16$
2. Answer true or false. $\int_0^2 (8x - x^3)\, dx = \int_0^2 (y - \sqrt[3]{y})\, dy$
3. Find the area enclosed by the curves $y = -x^5$, $y = -\sqrt[3]{x}$, $x = 0$ and $x = 1/2$.
   (a) 0.295
   (b) 0.315
   (c) 0.273
   (d) 0.279
4. Find the area enclosed by the curves $y = \sin(3x)$, $y = 2x$, $x = 0$ and $x = \pi$.
   (a) 4.27
   (b) 2.38
   (c) 9.32
   (d) 10.68
5. Find the area between the curves $y = |x-2|$, $y = \dfrac{x}{2} + 2$.
   (a) 3.0240
   (b) 12.000
   (c) 3.0251
   (d) 3.0262
6. Find the area between the curves $x = 2|y|$, $x = -2y + 4$ and $y = 0$.
   (a) 1
   (b) 2
   (c) 0.5
   (d) 0.3
7. Use a graphing utility to find the area of the region enclosed by the curves $y = x^3 - 2x^2 + 5x + 2$, $y = 0$, $x = 0$ and $x = 2$.
   (a) $34/3$
   (b) $37/3$
   (c) $38/3$
   (d) $40/3$
8. Use a graphing utility to find the area enclosed by the curves $y = -x^5$, $y = x^2$, $x = 0$ and $x = 3$.
   (a) 130.5
   (b) 120.75
   (c) 140.5
   (d) 125.25
9. Use a graphing utility to find the area enclosed by the curves $x = 2y^4$, $x = \sqrt{4y}$.
   (a) 2
   (b) 1
   (c) 0.76
   (d) 0.93
10. Answer true or false. The curves $y = x^2 + 5$ and $y = 6x$ intersect at $x = 1$ and $x = 2$.
11. Answer true or false. The curves $x = y^2 + 3$ and $x = 11y$ intersect at $y = 3$ and $y = 6$.
12. Answer true or false. The curves $y = \cos(x) - 1$ and $y = x^2$ intersect at $x = 0$ and $x = \pi$.
13. Answer true or false. The curves $y = 2\sin(\pi x/2)$ and $y = 2x^3$ intersect at $x = 0$ and $x = 1$.
14. Find a vertical line $x = k$ that divides the area enclosed by $y = -\sqrt{x}$, $y = 0$ and $x = 4$ into two equal areas.
    (a) $k = 4$
    (b) $k = 4^{2/3}$
    (c) $k = 4^{3/2}$
    (d) $k = 2$
15. Approximate the area of the region that lies below $y = 3\cos(x/3)$ and above $y = 0.3x$, where $0 \le x \le \pi$.
    (a) 6.314
    (b) 2.442
    (c) 0.558
    (d) 1.118

Section 6.2

1. Use the method of disks to find the volume of the solid that results by revolving the region enclosed by the curves $y=-x^3$, $x=-4$, $x=0$ and $y=0$ about the x-axis (round to the nearest whole number).

   (a) $7,353$

   (b) $3,677$

   (c) $14,706$

   (d) $46,201$

2. Use the method of disks to find the volume of the solid that results by revolving the region enclosed by the curves $y=\sqrt{\cos x}$, $x=0$, $x=\pi/2$ and $y=0$ about the x-axis.

   (a) $\pi/4$

   (b) 2π

   (c) $\pi/2$

   (d) π

3. Use the method of disks to find the volume of the solid that results by revolving the region enclosed by the curves $y=x^2-4$, $y=0$, and $x=0$ about the y-axis (round to the nearest whole number).

   (a) 17

   (b) 54

   (c) 64

   (d) 201

4. Use the method of disks to find the volume of the solid that results by revolving the region enclosed by the curves $y=x+e^x$, $y=0$, $x=0$ and $x=1$ about the x-axis. The approximate volume is

   (a) 5.53

   (b) 11.05

   (c) 17.37

   (d) 34.73

5. Answer true or false. The volume of the solid that results when the region enclosed by the curves $y=x^{10}$, $y=0$, $x=0$ and $x=2$ is revolved about the x-axis is given by $\int_0^2 \pi x^{20}\,dx$.

6. Answer true or false. The volume of the solid that results when the region enclosed by the curves $y=\sqrt[3]{x}$, $y=0$, $x=0$ and $x=3$ is revolved about the x-axis is given by $\left(\int_0^3 \pi\sqrt[3]{x}\,dx\right)^2$.

7. Use the method of disks to find the volume of the solid that results by revolving the region enclosed by the curves $y=-x^5$, $x=0$, $y=-1$ about the y-axis. The approximate volume is

   (a) 0.83

   (b) 2.62

   (c) 8.23

   (d) 2.24

8. Use the method of disks to find the volume of the solid that results by revolving the region enclosed by the curves $x=\sqrt{4y+16}$, $x=0$, and $y=-1$ about the y-axis. The approximate volume is

   (a) 56.5

   (b) 48.2

   (c) 15.3

   (d) 18.0

9. Use the method of disks to find the volume of the solid that results by revolving the region enclosed by the curves $x=y^2$, $x=-y+6$ about the y-axis. The approximate volume is

   (a) 20.83

   (b) 65.45

   (c) 523.60

   (d) $1,028.08$

10. Answer true or false. The volume of the solid that results when the region enclosed by the curves $x=y^6$ and $x=y^8$ is revolved about the y-axis is given by $\int_0^1 (y^6-y^8)^2\,dy$.

11. Find the volume of the solid whose base is enclosed by the circle $(x-2)^2+(y+3)^2=9$ and whose cross sections taken perpendicular to the base are semicircles. The approximate volume is

    (a) 113.10

    (b) 355.31

    (c) 56.5

    (d) 117.65

12. Answer true or false A right-circular cylinder of radius 8 cm contains a hollow sphere of radius 4 cm. If the cylinder is filled to a height of h cm with water and the sphere floats so that its highest point is 1 cm above the water level, there is $16\pi h - 8\pi/3$ cm^3 of water in the cylinder.

13. Use the method of disks to find the volume of the solid that results by revolving the region enclosed by the curves $y=\cos^6 x$, $x=2\pi$, and $x=5\pi/2$ about the x-axis. The approximate volume is

    (a) 0.35

    (b) 1.11

    (c) 0.49

    (d) 0.76

14. Use the method of washers to find the volume of the solid that results by revolving the region enclosed by the curves $y = -e^{2x}$, $x = 2$, and $y = -1$ about the x-axis (round to the nearest whole number).
    (a) $574,698$
    (b) $2,334$
    (c) 693
    (d) $2,178$
15. Answer true or false. The volume of the solid that results when the region enclosed by the curves $y = x^2$ and $x = y$ is revolved about $x = 1$ is $V = 0.133$ (rounded to 3 decimal places).

Section 6.3

1. Use cylindrical shells to find the volume of the solid when the region enclosed by $y = x^2$, $x = -2$, $x = -1$ and $y = 0$ is revolved about the y-axis.
    (a) $\dfrac{15\pi^2}{4}$
    (b) $\dfrac{15\pi}{4}$
    (c) $\dfrac{15\pi}{8}$
    (d) $\dfrac{15\pi}{2}$
2. Use cylindrical shells to find the volume of the solid when the region enclosed by $y = \sqrt{-x}$, $x = 0$, $x = -1$ and $y = 0$ is revolved about the y-axis.
    (a) 0.4π
    (b) 0.8π
    (c) 0.2π
    (d) $0.2\pi^2$
3. Use cylindrical shells to find the volume of the solid when the region enclosed by $y = \dfrac{3}{x^2}$, $x = 1$, $x = 2$ and $y = 0$ is revolved about the y-axis. The approximate volume is
    (a) 2.08π
    (b) 4.16π
    (c) $2.08\pi^2$
    (d) $4.16\pi^2$
4. Use cylindrical shells to find the volume of the solid when the region enclosed by $y = -x^{-3}$, $x = 1$, $x = 2$ and $y = 0$ is revolved about the y-axis.
    (a) π^2
    (b) 0.5π
    (c) π
    (d) $0.05\pi^2$
5. Use cylindrical shells to find the volume of the solid when the region enclosed by $y = -\sin(-x^2)$, $y = 0$, $x = 0$ and $x = 1$ is revolved about the y-axis. The approximate volume is
    (a) 0.560π
    (b) 0.520π
    (c) 0.460π
    (d) 0.500π
6. Use cylindrical shells to find the volume of the solid when the region enclosed by $y = 2e^{x^2}$, $x = 1$, $x = 2$ and $y = 0$ is revolved about the y-axis. The approximate volume is
    (a) 103.8π
    (b) 12.970π
    (c) 6.485π
    (d) 25.940π
7. Use cylindrical shells to find the volume of the solid when the region enclosed by $y = 4 - x$, $y = x$, and $y = 0$ is revolved about the y-axis.
    (a) 80π
    (b) 32π
    (c) 16π
    (d) 8π
8. Use cylindrical shells to find the volume of the solid when the region enclosed by $y = 2x^2 - 6x$ and $y = 0$ is revolved about the y-axis.
    (a) $\dfrac{27\pi}{2}$
    (b) 27π
    (c) $\dfrac{27\pi}{8}$
    (d) $\dfrac{27\pi}{4}$
9. Use cylindrical shells to find the volume of the solid when the region enclosed by $x = -y^2$, $x = 0$ and $y = -2$ is revolved about the x-axis.
    (a) 4
    (b) 4π
    (c) 8
    (d) 8π
10. Use cylindrical shells to find the volume of the solid when the region enclosed by $y = \sqrt[3]{8x}$, $x = 1$ and $y = 0$ is revolved about the x-axis.

(a) $\dfrac{19\pi}{20}$

(b) $\dfrac{12\pi}{5}$

(c) $\dfrac{10\pi}{3}$

(d) $\dfrac{8\pi}{5}$

11. Use cylindrical shells to find the volume of the solid when the region enclosed by $xy = 7$ and $x + y = -6$ is revolved about the x-axis. The approximate volume is

(a) 1.32π

(b) 16.4π

(c) 7.54π

(d) 65.6π

12. Use cylindrical shells to find the volume of the solid when the region enclosed by $y = -x^2$, $x = 1$, $x = 2$ and $y = 0$ is revolved about the line $x = 1$.

(a) $\dfrac{17\pi}{6}$

(b) $\dfrac{15\pi}{2}$

(c) $\dfrac{5\pi}{6}$

(d) $\dfrac{14\pi}{3}$

13. Use cylindrical shells to find the volume of the solid when the region enclosed by $y = x^2$, $x = 0$, $x = -2$ and $y = 0$ is revolved about the line $x = 1$.

(a) 8π

(b) $\dfrac{40\pi}{3}$

(c) $\dfrac{8\pi}{3}$

(d) $\dfrac{16\pi}{3}$

14. Answer true or false. The volume resulting from revolving the region enclosed by the semicircle $y = \sqrt{16 - x^2}$ about the x-axis is $\dfrac{32\pi}{3}$.

Section 6.4

1. Find the arc length of the curve $y = -2x^{3/2}$ from $x = 0$ to $x = 3$. The approximate arc length is

(a) 10.9

(b) 10.9π

(c) 6.8

(d) 6.8π

2. Find the arc length of the curve $y = \dfrac{1}{2}(x^2 + 3)^{3/2}$ from $x = 0$ to $x = 2$. The approximate arc length is

(a) 7.17

(b) 14.34

(c) 28.68

(d) 51.96

3. Answer true or false. The arc length of the curve $y = (x-2)^{5/2}$ from $x = 0$ to $x = 5$ is given by $\displaystyle\int_0^5 \sqrt{1 + (x-2)^5}\, dx$.

4. Answer true or false. The arc length of the curve $y = e^x + e^{2x}$ from $x = 0$ to $x = 4$ is given by $\displaystyle\int_0^4 \sqrt{1 + (e^{3x})^2}\, dx$.

5. The arc length of the curve $x = \dfrac{1}{6}(y^2 + 4)^{3/2}$ from $y = -1$ to $y = 0$ is

(a) $5/6$

(b) $7/6$

(c) $5/3$

(d) 4

6. Find the arc length of the parametric curve $x = \dfrac{3}{2}t^2$ and $y = t^3$ for $0 \le t \le 2$. The approximate arc length is

(a) 3.328

(b) 3.324

(c) 10.180

(d) 3.348

7. Find the arc length of the parametric curve $x = -\cos t$ and $y = \sin t$ for $0 \le t \le \pi/2$.

(a) $\pi/2$

(b) $\pi^2/4$

(c) $\sqrt{\pi}$

(d) π

8. Answer true or false. The arc length of the parametric curve $x = 3e^t$ and $y = e^t$ for $0 \le t \le 3$ is given by $\displaystyle\int_0^3 \sqrt{4e^t}\, dt$.

9. The arc length of the parametric curve $x = -\cos(2t)$, $y = \sin(2t)$ for $0 \le t \le 1$ is

(a) 2

(b) $\sqrt{2}$

(c) π

(d) 2π

10. Answer true or false. The arc length of the parametric curve $x = e^{3t}$ and $y = e^{3t}$ for $0 \le t \le 2$ is given by $\int_0^2 \sqrt{3}e^t\, dt$.
11. Use a CAS or a calculator with integration capabilities to approximate the arc length of the curve $y = \sin(-x)$ from $x = 0$ to $x = \pi/2$.
    (a) 1.43
    (b) 1.74
    (c) 1.86
    (d) 1.91
12. Use a CAS or a calculator with integration capabilities to approximate the arc length of the curve $x = \sin(-3y)$ from $y = 0$ to $y = \pi$.
    (a) 2.042
    (b) 6.987
    (c) 2.051
    (d) 2.916
13. Use a CAS or a calculator with integration capabilities to approximate the arc length of the curve $y = -xe^x$ from $x = 0$ to $x = 2$.
    (a) 21.02
    (b) 4.17
    (c) 15.04
    (d) 19.71
14. Answer true or false. The arc length of $y = x\cos x$ from $x = 0$ to $x = \pi$ can be approximated by a CAS or a calculator with integration capabilities to be 4.698.

Section 6.5

1. Find the area of the surface generated by revolving $y = -2x$, $0 \le x \le 1$ about the x-axis. The approximate surface area is
    (a) 4.47
    (b) 14.05
    (c) 28.10
    (d) 88.28
2. Find the area of the surface generated by revolving $y = \sqrt{1+x}$, $-1 \le x \le 0$ about the x-axis. The approximate surface area is
    (a) 4.47
    (b) 28.07
    (c) 5.33
    (d) 7.02
3. Find the area of the surface generated by revolving $x = \sqrt{y+1}$, $0 \le y \le 1$ about the y-axis. The approximate surface area is
    (a) 67.88
    (b) 3.44
    (c) 8.28
    (d) 21.60
4. Answer true or false. The area of the surface generated by revolving $x = \sqrt{3y}$, $1 \le y \le 5$ about the y-axis is given by $\int_1^5 2\pi y \left(1 + \frac{3}{4\sqrt{3x}}\right) dy$.
5. Answer true or false. The area of the surface generated by revolving $x = e^{y+2}$, $0 \le y \le 1$ about the y-axis is given by $2\pi e^2 \int_0^1 \sqrt{1 + e^{2y+4}}\, dy$.
6. Answer true or false. the area of the surface generated by revolving $x = \sin y$, $0 \le y \le \pi$ about the y-axis is given by $\int_0^\pi 2\pi y \sqrt{1 - \cos^2 x}\, dx$.
7. Use a CAS or a scientific calculator with numerical integration capabilities to approximate the area of the surface generated by revolving the curve $y = e^{x+1}$, $-1 \le x \le -0.5$ about the x-axis.
    (a) 18.54
    (b) 9.27
    (c) 1.48
    (d) 6.78
8. Use a CAS or a scientific calculator with numerical integration capabilities to approximate the area of the surface generated by revolving the curve $xy = 1$, $1 \le y \le 2$ about the x-axis.
    (a) 5.016
    (b) 5.394
    (c) 7.678
    (d) 10.502
9. Answer true or false. A CAS or a calculator with numerical integration capabilities can be used to approximate the area of the surface generated by revolving the curve $y = \cos x$, $0 \le x \le \pi/2$ about the x-axis to be 1.
10. Answer true or false. A CAS or a calculator with numerical integration capabilities can be used to approximate the area of the surface generated by revolving the curve $y = \sin x$, $0 \le x \le \pi/2$ about the x-axis to be 1.
11. Answer true or false. A CAS or a calculator with numerical integration capabilities can be used to approximate the area of the surface generated by revolving the curve $x = \cos y$, $0 \le y \le \pi/2$ about the y-axis to be 8.08.

12. Answer true or false. The area of the surface generated by revolving the parametric curve $x = t^2$ and $y = e^t$ for $0 \leq t \leq 1$ about the x-axis is given by $2\pi \int_0^1 e^t \sqrt{e^{2t} + 4t^2}\, dt$.
13. Answer true or false. The area of the surface generated by revolving the parametric curve $x = t^2$ and $y = e^t$ for $0 \leq t \leq 1$ about the y-axis is given by $2\pi \int_0^1 t^2 \sqrt{e^{2t} + 4t^2}\, dt$.
14. The area of the surface generated by revolving the parametric curve $x = 4\sin t$ and $y = 4\cos t$ for $0 \leq t \leq \pi$ about the x-axis is
    (a) $\frac{16\pi}{3}$
    (b) $\frac{8\pi}{3}$
    (c) $\frac{4\pi}{3}$
    (d) 64π

Section 6.6

1. Find the work done when a constant force of 20 lb in the positive x direction moves an object from $x = 3$ to $x = 4$ ft.
    (a) 20 ft-lb
    (b) 140 ft-lb
    (c) 40 ft-lb
    (d) 100 ft-lb
2. A spring whose natural length is 35 cm is stretched to a length of 40 cm by a 2 N force. Find the work done in stretching the spring.
    (a) 0.05 J
    (b) 0.4 J
    (c) 0.7 J
    (d) 0.3 J
3. Assuming that 20 J of work stretches a spring from its natural length of 60 cm to a length of 64 cm, find the spring constant in N/cm.
    (a) 4.03
    (b) 8.06
    (c) 125
    (d) 250
4. Answer true or false. Assume a spring is stretched from 100 cm to 140 cm by a force of 500 N. The work needed to do this is 200 J.
5. A cylindrical tank of radius 5 m and height 10 m is filled with a liquid whose density is 1.84 kg/m^3. How much work is needed to pump the liquid out of the tank?
    (a) $7,225.7$ J
    (b) $7,125.8$ J
    (c) $7,334.1$ J
    (d) $7,310.2$ J
6. Answer true or false. The amount of work needed to pump a liquid of density 0.95 kg/m^3 from a spherical tank of radius 4 m is $\int_0^8 0.95(8-x)\pi x^2\, dx$.
7. An object in deep space is initially considered to be stationary. If a force of 250 N acts on the object over a distance of 400 m, how much work is done on the object?
    (a) 0 J
    (b) $100,000$ J
    (c) $50,000$ J
    (d) $25,000$ J
8. Find the work done when a variable force of $F(x) = \frac{3}{x^2}$ N in the positive x-direction moves an object from $x = 2$ m to $x = 8$ m.
    (a) 5.64 J
    (b) 4.50 J
    (c) 2.25 J
    (d) 1.13 J
9. Find the work done when a variable force of $F(x) = \frac{1}{x^2}$ N in the positive x-direction moves an object from $x = -5$ m to $x = -4$ m.
    (a) 0.113 J
    (b) 1.00 J
    (c) 0.05 J
    (d) 0.25 J
10. Find the work done when a variable force of $F(x) = 30x$ N in the positive x-direction moves an object from $x = -4$ m to $x = 0$ m.
    (a) 240 J
    (b) 320 J
    (c) 80 J
    (d) 160 J
11. If the Coulomb force is proportional to x^{-2}, the work it does is proportional to
    (a) x^{-1}
    (b) x^{-3}
    (c) x

(d) x^{-2}

12. Answer true or false. It takes the same amount of work to move an object from $100,000$ km above the earth to $200,000$ km above the earth as it does to move the abject from $200,000$ km above the earth to $300,000$ km above the earth.
13. Answer true or false. It takes twice as much work to elevate an object to 120 m above the earth as it does to elevate the same object 60 m above the earth.
14. Answer true or false. It takes twice as much work to stretch a spring 100 cm as it does to stretch the same spring 50 cm.
15. A 1 kg object is moving at 10.0 m/s. If a force in the direction of motion does 40.0 J of work on the object, then what is the object's final speed?

    (a) 13.4 m/s

    (b) 5.5 m/s

    (c) 5.0 m/s

    (d) 11 m/s

Section 6.7

1. Consider the lamina bounded by the curves $y = x^2$, $x = a$ (with $a > 0$) in the first quadrant. If $\delta = x + 1$ then find an expression for the mass of the lamina in terms of a.

    (a) $\dfrac{1}{3}a^3$

    (b) $\dfrac{a^3}{3} + \dfrac{a^4}{4}$

    (c) $\dfrac{a^4}{4}(1 + (4/5)a)$

    (d) $\dfrac{a^5}{60}(6 + 5a)$

2. For the lamina in problem #1 find M_x in terms of a.

    (a) $\dfrac{1}{3}a^3$

    (b) $\dfrac{a^3}{3} + \dfrac{a^4}{4}$

    (c) $\dfrac{a^4}{4}(1 + (4/5)a)$

    (d) $\dfrac{a^5}{60}(6 + 5a)$

3. For the lamina in problem #1 find M_y in terms of a.

    (a) $\dfrac{1}{3}a^3$

    (b) $\dfrac{a^3}{3} + \dfrac{a^4}{4}$

    (c) $\dfrac{a^4}{4}(1 + (4/5)a)$

    (d) $\dfrac{a^5}{60}(6 + 5a)$

4. A triangular region in the first quadrant is bounded by the x-axis, y-axis, and the line between the points $(0, a)$ and $(b, 0)$ where $a, b > 0$. Find the centroid of this region.

    (a) $\left(\dfrac{ab}{2}, \dfrac{ab}{2}\right)$

    (b) $\left(\dfrac{b}{2}, \dfrac{a}{2}\right)$

    (c) $\left(\dfrac{a^2b}{6}, \dfrac{ab^2}{6}\right)$

    (d) $\left(\dfrac{b}{3}, \dfrac{a}{3}\right)$

5. Consider the region R in the first quadrant that is bounded by $y = 4 - x^2$ with a constant density δ. If the centroid of the region is located at $(\bar{x}, \bar{y})$, then which of the following statements is true?

    (a) $M = \frac{8}{3}\delta$

    (b) $\bar{y} = \frac{8}{5}$

    (c) $\bar{x} = \frac{128}{15}$

    (d) $M_x = 4\delta$

6. Consider the region in the first quadrant that is bounded by the circle $x^2 + y^2 = r^2$ with a density of $\delta = 1$. Which of the following statements is true?

    (a) $M = \pi r^2$

    (b) $M_y = \pi r/2$

    (c) $M_x = M_y$

    (d) The centroid of the region cannot be found.

7. Consider the lamina bounded by the curves $x = y^2 - 4$ and $y = x/3$ with a constant density δ. Which of the following expressions represents the mass of the region?

    (a) $\displaystyle\int_{-3}^{12} \delta \cdot (\sqrt{x+4} - x/3)\, dx$

    (b) $\displaystyle\int_{-1}^{4} \delta \cdot (3y - y^2 + 4)\, dy$

    (c) $\displaystyle\int_{-1}^{4} \delta \cdot y \cdot (3y - y^2 + 4)\, dy$

    (d) None of the above

8. For the region described in Problem 7, which of the the following expressions represents the moment about the x-axis, M_x.

    (a) $\displaystyle\int_{-1}^{4} \delta \cdot (3y - y^2 + 4)\, dy$

(b) $\int_{-4}^{12} \delta \cdot x \cdot (\sqrt{x+4} - x/3)\, dx$

(c) $\int_{-4}^{12} \frac{\delta}{2} \left(x+4-\frac{x^2}{9} \right) dx$

(d) None of the above

9. Answer true or false. For the region described in Problem 7, the x-coordinate of the centroid of the region is negative.

10. Find the centroid of the region bounded by the curves $y = \sqrt{x}$, $y = 1$, and $x = 4$.

    (a) $(2.94, 1.35)$

    (b) $(1.35, 2.94)$

    (c) $(2.25, 4.9)$

    (d) $(4.9, 2.25)$

11. The quadrilateral $ABCD$ region has corners at the points $A(1,5)$, $B(8,4)$, $C(6,0)$, $D(2,2)$. Find the centroid of the region.

    (a) $(9/2, 17/6)$

    (b) $(9/2, 5/2)$

    (c) $(44/9, 5/2)$

    (d) $(17/4, 11/4)$

12. A laminar region is bounded by the curves $x = 1$, $x = 10$, $y = 0$, and $y = 1/x$ with $\delta = x^2 + 1$. Find the center of gravity of the region.

    (a) $(5.17581, 0.14168)$

    (b) $(6.60199, 0.09556)$

    (c) $(6.72727, 0.09091)$

    (d) $(3.90865, 0.19543)$

Section 6.8

1. A flat rectangular plate is submerged horizontally in water to a depth of 6.0 ft. If the top surface of the plate has an area of 50 ft^2, and the liquid in which it is submerged is water, then find the force on the top of the plate. Neglect the effect of the atmosphere above the liquid. (The density of water is 62.4 lb/ft^3.

    (a) 300 lb

    (b) 33.4 lb

    (c) $2,080$ lb

    (d) $18,720$ lb

2. Find the force (in N) on the top of a submerged object if its surface is 10.0 m^2 and the pressure acting on it is 3.2×10^5 Pa. Neglect the effect of the atmosphere above the liquid.

    (a) 3.7×10^4 N

    (b) 3.2×10^6 N

    (c) 6.4×10^6 N

    (d) 1.7×10^6 N

3. Find the force on a 100 ft wide by 5 ft deep wall of a swimming pool filled with water. Neglect the effect of the atmosphere above the liquid. (The density of water is 62.4 lb/ft^3.)

    (a) $78,000$ lb

    (b) $124,800$ lb

    (c) $62,400$ lb

    (d) 624 lb

4. Answer true or false. The force a liquid of density ρ exerts on an equilateral triangle with edges h in length submerged point down is given by $\int_0^h \frac{\rho}{3} x^2\, dx$.

5. A right triangle is submerged vertically with one side at the surface in a liquid of density ρ. The triangle has a leg that is 20 m long located at the surface and a leg 10 m long straight down. Find the force exerted on the triangular surface, in terms of density. Neglect the effect of the atmosphere above the liquid.

    (a) 677ρ N

    (b) 500ρ N

    (c) 600ρ N

    (d) 200ρ N

6. Answer true or false. A glass circular window on the side of a submarine has the same force acting on the top half as on the bottom half.

7. Find the force on a 30 ft^2 horizontal surface 20 ft deep in water. Neglect the effect of the atmosphere above the liquid. (The density of water is 62.4 lb/ft^3.)

    (a) 600 lb

    (b) $37,440$ lb

    (c) $1,200$ lb

    (d) $30,000$ lb

8. Answer true or false. A flat sheet of material is submerged vertically in water. The force acting on each side must be the same.

9. Answer true or false. If a submerged horizontal object is elevated to half its original depth, the force exerted on the top of the object will be half the force originally exerted on the object. Assume there is a vacuum above the liquid surface.

10. Answer true or false. If a square, flat surface is suspended vertically in water and its center is 20 m deep, the force on the object will double if the object is relocated to a depth of 40 m. Neglect the effect of the atmosphere above the liquid.

11. Answer true or false. The force on a semicircular, vertical wall with top d is given by $\int_0^{1/2} 2\rho x\sqrt{\frac{d^2}{4}-x^2}\,dx$.
12. Answer true or false. The force exerted by water on a surface of a square, vertical plate with edges of 3 m if it is suspended with its top 2 m below the surface is 18 lb. (The density of water is 62.4 lb/ft^3.)
13. Answer true or false. If a submerged rectangle is rotated 90° about an axis through its center and perpendicular to its surface, the force exerted on one side of it will be the same, provided the entire rectangle remains submerged.

Section 6.9

1. Evaluate $\sinh(7)$.
   (a) Not defined.
   (b) 551.1614
   (c) 548.3161
   (d) 549.4283
2. Evaluate $\cosh^{-1}(2)$.
   (a) 1.3170
   (b) 1.3165
   (c) 1.3152
   (d) 1.3174
3. Find dy/dx if $y=\sinh(5x+1)$.
   (a) $(5x+1)\cosh(5x+1)$
   (b) $5\cosh(5x+1)$
   (c) $-(5x+1)\cosh(5x+1)$
   (d) $-5\cosh(5x+1)$
4. Find dy/dx if $y=\sinh(3x^2)$.
   (a) $6x\cosh(3x^2)$
   (b) $-6x\cosh(3x^2)$
   (c) $6\cosh(6x)$
   (d) $-6\cosh(6x)$
5. Find dy/dx if $y=2\sqrt{\operatorname{sech}(x+5)-x^3}$.
   (a) $\dfrac{-\operatorname{sech}(x+5)\tanh(x+5)-3x^2}{\sqrt{\operatorname{sech}(x+5)-x^3}}$
   (b) $\dfrac{(x+5)\cosh(x+5)-3x^2}{\sqrt{\sinh(x+5)-x^3}}$
   (c) $\dfrac{-\cosh(x+5)+3x^2}{\sqrt{\sinh(x+5)-x^3}}$
   (d) $\dfrac{\operatorname{sech}(x+5)\tanh(x+5)+3x^2}{\sqrt{\operatorname{sech}(x+5)-x^3}}$
6. $\int \sinh(3x+6)\,dx =$
   (a) $3\cosh(3x+6)+C$
   (b) $\frac{1}{3}\cosh(3x+6)+C$
   (c) $-3\cosh(3x+6)+C$
   (d) $-\frac{1}{3}\cosh(3x+6)+C$
7. $\int \cosh^7 x\sinh x\,dx =$
   (a) $\frac{1}{8}\cosh^8 x+C$
   (b) $8\cosh^8 x+C$
   (c) $7\cosh^6 x+C$
   (d) $\frac{1}{6}\cosh^6 x+C$
8. $\int \sinh^9 x\cosh x\,dx =$
   (a) $\frac{1}{10}\sinh^{10} x+C$
   (b) $10\sinh^{10} x+C$
   (c) $9\sinh^8 x+C$
   (d) $\frac{1}{8}\sinh^8 x+C$
9. Find dy/dx if $y=\sinh^{-1}\left(\frac{x}{6}\right)$.
   (a) $\dfrac{1}{\sqrt{36+x^2}}$
   (b) $\dfrac{1}{6\sqrt{36+x^2}}$
   (c) $\dfrac{1}{\sqrt{36-x^2}}$
   (d) $\dfrac{1}{6\sqrt{36-x^2}}$
10. Answer true or false. If $y=-\coth^{-1}(x+3)$ when $|x|>0$, then $dy/dx=-\dfrac{1}{x^2+6x+8}$.
11. $\int \dfrac{dx}{\sqrt{1+16x^2}} =$
   (a) $\frac{1}{4}\sinh^{-1}(4x)+C$
   (b) $\frac{1}{4}\coth^{-1}(4x)+C$
   (c) $\frac{1}{4}\cosh^{-1}(4x)+C$

(d) $\dfrac{1}{4}\tanh^{-1}(4x)+C$

12. Answer true or false. $\displaystyle\int \frac{4dx}{1+e^{2x}} = 4\sinh^{-1}(e^x)+C$
13. Answer true or false. $\displaystyle\int \frac{e^x dx}{\sqrt{1+e^{2x}}} = \sinh^{-1}(e^{2x})+C$
14. Answer true or false. $\lim_{x\to\infty}(\cosh x)^2 = 0$.
15. Answer true or false. $\lim_{x\to-\infty}(\coth x)^2 = 1$.

Chapter 6 Test

1. Find the area of the region enclosed by $y = x^2$ and $y = x$ by integrating with respect to x.
   (a) $1/6$
   (b) 1
   (c) $1/4$
   (d) $1/16$
2. Find the area of the region enclosed by $y = \cos(x-\pi/2)$, $y=-x$, $x=0$ and $x=\pi/2$. The approximate area is
   (a) 1.1169
   (b) 2.2337
   (c) 4.4674
   (d) 1
3. Find the volume of the solid that results when the region enclosed by the curves $y=\sqrt{-\sin(-x)}$, $y=0$ and $x=\pi/4$ is revolved about the x-axis. The approximate volume is
   (a) 0.143
   (b) 0.920
   (c) 1.408
   (d) 2.816
4. Find the volume of the solid that results when the region enclosed by the curves $x=-e^y$, $x=-1$ and $y=1$ is revolved about the y-axis. The approximate volume is
   (a) 6.894
   (b) 3.195
   (c) 10.205
   (d) 32.060
5. Answer true or false. Cylindrical shells can be used to find the volume of the solid when the region enclosed by $y=\sqrt[3]{x}$, $x=-3$, $x=0$ and $y=0$ is revolved about the y-axis and the volume of the solid is 5.563π.
6. Answer true or false. Cylindrical shells can be used to find the volume of the solid when the region enclosed by $x=y^2$, $x=0$ and $y=-2$ is revolved about the x-axis and the volume of the solid is 4π.
7. Answer true or false. The arc length of $y=\cos(-x)$ from $x=0$ to $x=\pi/2$ is 1.
8. Answer true or false. The arc length of the parametric curve $x=\sin t$ and $y=-\cos t$, $0\le t\le \pi/2$ is $\pi/2$.
9. Answer true or false. The surface area of the curve $y=\sin(x+\pi)$, $-\pi\le y\le 0$ revolved about the x-axis is given by $\displaystyle\int_0^{\pi} 2\pi x\sqrt{1+\sin^2(x+\pi)}\,dx$.
10. Use a CAS to find the surface area of the solid that results when the curve $y=-e^x$, $0\le x\le 0.5$ is revolved about the x-axis. The approximate surface area is
    (a) 18.54
    (b) 9.27
    (c) 1.48
    (d) 6.78
11. Assume a spring whose natural length is 2.0 m is stretched 0.8 m by a 150 N force. How much work is done in stretching the spring?
    (a) 60 J
    (b) 6,120 J
    (c) 6,000 J
    (d) 240 J
12. Find the work done when a constant force $F(x) = 15$ N in the positive x-direction moves an object from $x=4$ m to 10 m.
    (a) 45 J
    (b) 90 J
    (c) 180 J
    (d) 150 J
13. Find the work done when a variable force of $F(x)=\dfrac{4}{x^2}$ N in the positive x-direction moves an object from $x=1$ m to $x=3$ m.
    (a) 0 J
    (b) 2.67 J
    (c) 0.6 J
    (d) 1.79 J
14. Answer true or false. A semicircular wall 20 ft across at the top forms one end of a tank. The total force exerted on this wall by a liquid that fills the tank is 24,800 lb. Ignore the force of air above the liquid. (The density of the liquid is 124.8 lb/ft^3.)

15. A horizontal table top is submerged in 10 ft of water. If the dimensions of the table are 6 ft by 1 ft, find the force on the top of the table that exceeds the force that would be exerted by the atmosphere if the table were at the surface of the water. (The density of water is 62.4 lb/ft^3.)

    (a) $3,744$ lb

    (b) $1,872$ lb

    (c) $4,000$ lb

    (d) 60 lb

16. Find dy/dx if $y = \tanh(x^5)$.

    (a) $5x^4 \text{sech}^2(x^5)$

    (b) $-5x^4 \text{sech}^2(x^5)$

    (c) $5x^4 \tanh(x^5)$

    (d) $\text{sech}^2(5x^4)$

17. $\int \tanh^6 x \text{sech}^2 x \, dx =$

    (a) $4\tanh^4 x + C$

    (b) $5\tanh^6 x + C$

    (c) $6\tanh^6 x + C$

    (d) $\frac{1}{7}\tanh^7 x + C$

18. Answer true or false. $\int \frac{4dx}{\sqrt{e^{2x}-1}} = 4\cosh^{-1}(e^x)$

19. Answer true or false. $\lim_{x\to\infty} (\coth x)^2 = 1$.

20. Evaluate $\cosh(1)$.

    (a) 1.543

    (b) 1.551

    (c) 1.562

    (d) 1.580

21. Find the centroid of the region bounded between the curves $y = |x|$ and $x + 2y = 3$.

    (a) $(-4/3, 2/3)$

    (b) $(-2/3, 4/3)$

    (c) $(-3/2, 4/3)$

    (d) $(-3/2, 2)$

Chapter 6: Answers to Sample Tests

Section 6.1

1. a	2. false	3. a	4. c	5. b	6. b	7. c	8. a
9. d	10. false	11. false	12. false	13. true	14. b	15. a	

Section 6.2

1. a	2. d	3. b	4. c	5. true	6. false	7. d	8. a
9. c	10. false	11. c	12. false	13. b	14. b	15. false	

Section 6.3

1. d	2. b	3. b	4. c	5. c	6. a	7. a	8. b
9. d	10. b	11. c	12. a	13. b	14. false		

Section 6.4

1. a	2. a	3. false	4. false	5. b	6. c	7. a	8. false
9. a	10. false	11. d	12. b	13. c	14. false		

Section 6.5

1. b	2. c	3. c	4. false	5. false	6. false	7. d	8. d
9. false	10. false	11. false	12. true	13. true	14. d		

Section 6.6

1. a	2. a	3. d	4. false	5. a	6. false	7. b	8. d
9. c	10. a	11. a	12. false	13. false	14. false	15. a	

Section 6.7

1. b	2. d	3. c	4. d	5. b	6. c	7. b	8. d
9. false	10. a	11. a	12. b				

Section 6.8

1. d	2. b	3. a	4. false	5. a	6. false	7. b	8. true
9. true	10. true	11. false	12. false	13. true			

Section 6.9

1. c	2. a	3. b	4. a	5. a	6. b	7. a	8. a
9. a	10. false	11. a	12. false	13. false	14. false	15. false	

Chapter 6 Test

1. a	2. b	3. b	4. a	5. false	6. false	7. false	8. true
9. false	10. d	11. a	12. b	13. b	14. false	15. a	16. a
17. d	18. false	19. true	20. a	21. b			