Chapter 8: Mathematical Modeling with Differential Equations

Summary: This chapter brings together the two important ideas of differentiation and integration of functions. These ideas are related by looking at equations that involve both a function, $y(x)$ and some of its derivatives, (i.e., $y'(x)$, $y''(x)$, etc.) which may be solved in some cases using integration. Topics discussed in this chapter include how to solve first order differential equations, how to use differential equations to model various physical situations, and how to find the unique solution of a differential equation that is paired with an initial condition.

OBJECTIVES: After reading and working through this chapter you should be able to do the following:

1. Determine the order of a differential equation (§8.1).
2. Determine if a given function is a solution of a differential equation (§8.1).
3. Solve a first order separable differential equation (§8.2).
4. Draw a slope field or direction field for a differential equation (§8.3).
5. Use Euler's method to approximate the solution of a differential equation (§8.3).
6. Solve a first order linear differential equation (§8.4).
7. Model various applications using first order differential equations (§8.1, §8.2, and §8.4).

8.1 Modeling With Differential Equations

PURPOSE: To define what a differential equation is and to solve some first order differential equations.

This section introduces what differential equations are and what it means for a function to solve a differential equation. Two types of differential equations are discussed: first order linear equations and separable equations.

CAUTION: Differential equations will involve some unknown function $y = y(t)$. Usually this function is just written as y although it is still a function of some independent variable.

A first-order differential equation is an equation that involves only y, its derivative, y' and an independent variable t. Sometimes, the independent variable will not occur explicitly in the equation. For example, $y' = y^2$, really could be written as

$$y'(t) = y(t)^2.$$

solution
general solution
initial-value problem (IVP)
initial conditions

A function, $y = \phi(t)$, is a **solution** of the differential equation if an equality results (i.e., both sides of the equation are equal) when the function is inserted into the differential equation. A **general solution** of a differential equation involves a constant of integration that allows for an infinite number of different functions to solve the differential equation. An **Initial-Value Problem (IVP)** is a differential equation that is paired with some initial condition. A solution of an IVP must solve the differential equation and also satisfy the initial condition(s). **Initial conditions** are substituted into the general solution in order to determine a unique value for the constant of integration.

Checklist of Key Ideas:

- ☐ differential equation
- ☐ order
- ☐ general solution
- ☐ integral curves
- ☐ initial-value problems
- ☐ initial conditions
- ☐ logistic growth
- ☐ spread of disease

8.2 Separation of Variables

Purpose: To solve separable differential equations.

separable equation

Some first-order differential equations are separable. A **separable equation** can be written in the form

$$H(y)y' = F(t) \longrightarrow H(y)dy = F(t)dt.$$

Then to find y, just integrate both sides. The left side should be integrated with respect to y and the right side should be integrated with respect to the independent variable (t in this case). After doing this, it may or may not be possible to solve explicitly for y.

CAUTION: Some separable differential equations do not appear to have an independent variable. For example, $y' = y^2$, really should be thought of as, $y'(t) = y(t)^2$.

Many applications can be modeled by first-order differential equations. Some of the applications that are discussed in this section are (1) the exponential growth or decay of some substance or population, (2) determining the amount of a drug present in the bloodstream at some time, t, (3) the spread of disease over time, and (4) continuous compounding. The equation that is most often used in these models is the following initial-value problem:

$$y' = ky, \; y(t_0) = y_0$$

This equation relates to exponential growth and decay which shows up in a surprising number of different applications.

growth vs. decay

In this differential equation, when $k > 0$ then it models **growth** and when $k < 0$ then it models **decay**. The general solution of the differential equation $y' = ky$ is always of the form

$$y = Ce^{kt}.$$

$y' = ky$ is solved either as a first-order linear equation or as a separable equation

The constant C is determined by the initial conditions at $t = t_0$. If $t = t_0$ then the solution becomes

$$y = y_0 e^{kt}$$

For all of the application problems that involve this differential equation, a key item is being able to find the growth/decay constant, k. A typical problem usually has the following format:

1. Given: initial condition at t_0.
2. Given: a value for y at some other time.
3. Find: the value of y at some t.

The method of solution typically has the following structure:

1. Use the initial condition to find C.
2. Find k using the other specified value for y.
3. Find the value of y at the time in question.

half-life
doubling time

Finding k often involves using what is called the **half-life** or the **doubling time**. Each of these are simply ways of specifying a value for y at some given time. The half-life gives the time after which only half of the original amount is left. Half-life occurs in decay problems ($k < 0$). The doubling time indicates how long it takes for the amount present to become twice the original amount. Doubling time occurs in growth problems (i.e., $k > 0$). Generally if $t = T$ is the doubling time then the growth constant k is given by

$$k = \frac{\ln 2}{T}.$$

On the other hand, if $t = T$ is the half-life, the decay constant k is given by

$$k = \frac{\ln(1/2)}{T} = -\frac{\ln 2}{T}.$$

IDEA: Half-life or doubling time are calculated the same way. For the differential equation $y' = ky$ if

1. $y_0 = y(t_0) \leftarrow$ the original amount
2. $y_a = y(t_a) \leftarrow$ amount at some later time

then the growth/decay constant is $k = \dfrac{\ln(y_a/y_0)}{t_a}$.

Checklist of Key Ideas:

- ☐ first order separable equations
- ☐ method of separation of variables
- ☐ exponential growth or decay models
- ☐ k, the growth or decay constant
- ☐ general solution of $y' = ky$
- ☐ relative growth or decay rate
- ☐ doubling time or half-life
- ☐ radioactive decay
- ☐ carbon-14 dating

8.3 Slope Fields; Euler's Method

PURPOSE: To introduce a method for visualizing the behavior of solutions to first-order differential equations.

A key feature of any differential equation is that it involves the derivative (or derivatives) of some (unknown) function. The derivative itself gives a way for finding the slope of a function at a point. A **slope field** uses a differential equation to find the slopes of possible solutions to the differential equation at many different points. This can often provide useful information about the differential equation and how it will behave.

slope field

IDEA: The equation $y' = f(x,y)$ allows the slope of y to be calculated at some point (x,y) by evaluating $f(x,y)$ at the point.

Euler's method uses the idea of the derivative representing the slope to approximate a solution of an initial-value problem that involves a first order differential equation.

Euler's method

IDEA: Euler's method for solving $y' = f(x,y)$ with $y(x_0) = y_0$:

$$y_{n+1} = y_n + \Delta x f(x_n, y_n)$$

Euler's method is closely related to the left rectangle method for approximating the value of an integral. The fundamental theorem of calculus says

$$\int_{x_n}^{x_{n+1}} y'\,dx = y(x_{n+1}) - y(x_n).$$

Then Euler's method makes two approximations: one for the left side of this equation and one for the right. The integral is approximated by a left rectangle. The value of the derivative at the left end of the interval is $f(x_n, y_n)$. The width of the interval is $x_{n+1} - x_n = \Delta x$. So the integral is approximately equal to the area of the left rectangle which is given by $\Delta x \cdot f(x_n, y_n)$. Then the right hand side of the equation is replaced with the approximate values of y. For example, $y_{n+1} - y_n \approx y(x_{n+1}) - y(x_n)$. So the fundamental theorem of calculus becomes

$$\Delta x f(x_n, y_n) = y_{n+1} - y_n.$$

When this is solved for y_{n+1}, this gives Euler's Method.

IDEA: Typically $x_i = x_0 + i\Delta x$ and y_i is the approximate value of $y(x_i)$ in Euler's method.

Euler's method is easily implemented. All that is needed is a starting value for $x = x_0$ and $y = y_0$ which are given in an IVP. Then a value for Δx is picked and the next value y_1 is approximated. Then $x_1 = x_0 + \Delta x$ and y_1 are used to find y_2 and so forth. For example if $y(0) = 1$ then $x_0 = 0$ and $y_0 = 1$. Then to approximate $y(1)$, a choice for Δx needs to be made. If $\Delta x = 0.5$ then $x_1 = x_0 + \Delta x = 0.5$ and $x_2 = x_1 + \Delta x = 1$ so $y(1)$ would be approximated by y_2 (i.e., $y_2 \approx y(x_2) = y(1)$).

Checklist of Key Ideas:

- ☐ slope field or direction field
- ☐ Euler's method
- ☐ increment or step-size, Δx
- ☐ absolute and percentage error

8.4 First-Order Differential Equations and Applications

PURPOSE: To solve linear first order differential equations and their applications.

first-order linear

First-order linear differential equations are differential equations that can be written in the following form.

$$y' + p(t)y = g(t)$$

linear vs. nonlinear

Anything that cannot be written in this form is **nonlinear**. The equation, $y' = y^2$, is first-order but it is nonlinear since it cannot be written in linear form.

method of integrating factor

First-order linear differential equations can be solved using the **method of integrating factor**. Here is an outline for this solution method:

1. write the D.E. as $y' + py = g$
2. find the integrating factor $\mu(t) = e^{\int p(t)dt}$
3. multiply by μ to obtain
$$\frac{d}{dt}\left[y \cdot \mu\right] = g \cdot \mu$$
4. Integrate both sides (with respect to t) to obtain
$$y \cdot \mu = G(t) + C$$
where $G(t) = \int g \cdot \mu \; dt$
5. Solve for y by dividing by μ

mixing problems

Mixing problems always have the following format:

$$\frac{dy}{dt} = \text{rate in} - \text{rate out},$$

where $y(t)$ represents the amount of some substance in a mixing tank. A common mistake is to assume that $y(t)$ represents the liquid in the tank or that $y(t)$ is a concentration.

CAUTION: The function $y(t)$ represents an amount, not a concentration.

This is not the case. The function $y(t)$ usually will have units that represent some amount. The differential equation is often easier to write down by keeping track of the units. All three terms, dy/dt, the “rate in”, and the “rate out” must have the same units. For example,

$$\text{rate in} = (\text{concentration in}) \times (\text{flowrate in}).$$

The velocity of a **falling body** can be modeled by the differential equation

falling body

$$\frac{dv}{dt} + \frac{c}{m}v = -g, \qquad v(0) = v_0$$

where m is the mass of the object, g is the gravitational constant, and c is a constant of proportionality for the drag force due to wind resistance. Note that this differential equation assumes that down is the negative direction. This is a linear differential equation that can be solved by the method of integrating factor. After solving the differential equation, it can be seen that the speed of the object after a long period of time will approach a **terminal speed** of $v_t = \dfrac{mg}{c}$.

terminal speed

Checklist of Key Ideas:

- first order linear equations
- integrating factor and the method of integrating factors
- mixing problems
- modeling free fall with air resistance
- terminal or limiting velocity

Chapter 8 Sample Tests

Section 8.1

1. State the order of the differential equation $3y'' + 7y = 0$.
   (a) 0
   (b) 1
   (c) 2
   (d) 3
2. State the order of the differential equation $y' - 5y^2 = 0$.
   (a) 0
   (b) 1
   (c) 2
   (d) 3
3. Answer true or false. The differential equation $y' - 5y = 0$ has a general solution of $y = Ce^{-5t}$.
4. Answer true or false. The differential equation $(3+x)\dfrac{dy}{dx} = 1$ is solved by $y = \ln|3+x| + C$ when $x \geq 0$.
5. Find the unique solution of the initial-value problem $\dfrac{dy}{dt} = t^3$, $y(0) = 4$.
   (a) $y = \dfrac{1}{4}t^4 + 4$
   (b) $y = 4t^4 + 4$
   (c) $y = 0$
   (d) $y = \dfrac{1}{2}t^2 + 4$
6. Find the general solution of the differential equation $\dfrac{dy}{dt} = t^{1/5}$.
   (a) $y = t^{6/5} + C$
   (b) $y = 2t^{6/5} + C$
   (c) $y = \dfrac{6}{5}t^{6/5} + C$
   (d) $y = \dfrac{5}{6}t^{6/5} + C$
7. If $y = Ce^t + 5t$ is the general solution to a differential equation with $y(0) = 5$ then what is the value of the constant C?
   (a) 0
   (b) -1
   (c) 1
   (d) 5
8. The function $y = Ce^{5t} + 5t$ is the general solution to which of the following differential equations?
   (a) $y' - 5y = -25t + 5$
   (b) $y' - 5y = 5t$
   (c) $y' + 5y = 25t - 5$
   (d) $y' + 5y = 5t$
9. Which of the following differential equations models logistic growth?
   (a) $y' = ky$
   (b) $y' = ky^2$
   (c) $y' = k\left(1 - \dfrac{y}{L}\right)y$
   (d) $y' = ky(1 - y^2)$
10. Find the unique solution of the differential equation $y'' = -3x$ with $y(0) = 2$ and $y'(0) = -5$.
   (a) $y = -\dfrac{1}{2}x^3 - 5x + 2$
   (b) $y = -5x + 2$
   (c) $y = -\dfrac{3}{2}x^2 - 5x + 2$
   (d) $y = -3x^2 - 5x + 2$

Section 8.2

1. Find the general solution of the differential equation $\dfrac{dy}{dt} + y^3 = 0$.
   (a) $y = e^{3t} + C$
   (b) $y = e^t + C$
   (c) $y = \frac{1}{3}t^{1/3} + C$
   (d) $y = \pm(2t + C)^{-1/2}$
2. Find the unique solution of the initial-value problem $\dfrac{dy}{dt} = y^2$, $y(1) = -1$.
   (a) $3y - y^3 - 2 = 0$
   (b) $y + y^3 + 2 = 0$
   (c) $y = \dfrac{1}{3}t^{1/3} + 2$
   (d) $y = -\dfrac{1}{t}$
3. Find the general solution of the differential equation $\dfrac{2x}{y} = y'$.
   (a) $y = \pm(2x^2 + C)^{1/2}$
   (b) $y = \pm\sqrt{2}x + C$

(c) $y = Ce^{2x}$

(d) $y = e^{2Cx}$

4. Find the unique solution of the initial-value problem $\frac{x}{y} = y'$, $y(1) = 3$.

(a) $y = x + 2$

(b) $y = 2x + 1$

(c) $y = \sqrt{x^2 + 8}$

(d) $y = 3e^{x-1}$

5. Answer true or false. Suppose that a quantity $y = y(t)$ changes in such a way that $dy/dt = ky^{1/5}$, where $k > 0$. It can be said that the rate of change of the quantity is proportional to the fifth root of the amount present.

6. Answer true or false. Suppose that a quantity $y = y(t)$ changes in such a way that $dy/dt = k\sqrt{y}$, where $k > 0$. It can be said that y increases at a rate that is proportional to the square root of the time, t.

7. If an initial population of P_o bacteria is growing at a rate of 3% per hour, then the number of bacteria present t hours later is

(a) $P_o(1 + 0.03t)$

(b) $P_o e^{0.03t}$

(c) $P_o e^{1.03t}$

(d) $P_o(1.03)^t$

8. A given radioactive substance has a half-life of 151 years. Find a formula for the amount of substance, y, that is present at time, t if 500 g of the substance are present initially.

(a) $y = 500e^{-0.00459t}$

(b) $y = 500e^{-0.693t}$

(c) $y = 500e^{0.717t}$

(d) $y = 500e^{-0.011t}$

9. If 400 g of a radioactive substance decays to 60 g in 12 years, then find the half-life of the substance.

(a) 7.06 years

(b) 4.38 years

(c) 1.9 years

(d) 0.15 years

10. A particular radioactive substance is found to decay according to the equation $y' = ky^2$. If 100 g of the substance are initially present, then what is the value of k if 50 g are present after 4 days?

(a) $k = -0.173$ (g·days)$^{-1}$

(b) $k = -0.0025$ (g·days)$^{-1}$

(c) $k = -0.347$ (g·days)$^{-1}$

(d) $k = -0.0433$ (g·days)$^{-1}$

11. If $y = y_o e^{kt}$ where $k > 0$ and $y_o > 0$, then the quantity that is being modeled

(a) is increasing.

(b) is decreasing.

(c) is constant.

(d) undetermined; more information is needed.

12. If a certain population, y, is growing exponentially according to $y = y_o e^{kt}$ then determine the value of k if the population doubles in $T = 10$ months.

(a) $k = 10\ln 2$ (months)$^{-1}$

(b) $k = \frac{y_o \ln 2}{10}$ (months)$^{-1}$

(c) $k = \frac{\ln 2}{10}$ (months)$^{-1}$

(d) $k = 10y_o \ln(1/2)$ (months)$^{-1}$

13. Answer true or false. If $y(0) = 20$ and the substance represented increases at a rate of 8%, then $y = 20(0.08)^t$.

14. Answer true or false. If $y(0) = 40$ and the substance that is represented decreases at a rate of 16%, then $y = 40(0.16)^t$.

Section 8.3

1. If $y' = -x + 5y$, then the slope of the direction field at $(1,2)$ is

(a) 9

(b) -9

(c) $1/9$

(d) $-1/9$

2. If $y' = \cos(xy)$, then the slope of the direction field at $(0,3)$ is

(a) 1

(b) π

(c) -1

(d) 0

3. If $y' = \cos(3xy)$, then the slope of the direction field at $(5,0)$ is

(a) 1

(b) π

(c) -1

(d) 0

4. If $y' = x\cos y$, then the slope of the direction field at $(7,0)$ is

(a) 7

(b) -7

(c) 0

(d) 1

5. If $y' = ye^x$, then the slope of the direction field at $(6,0)$ is

(a) $6e^6$

(b) e^6

(c) 0

(d) 6

6. If $y' = 4x - 4y$, then the slope of the direction field at $(1,1)$ is

(a) 4

(b) -8

(c) 8

(d) 0

7. If $y' = \dfrac{x}{3y}$, then what is the slope of the direction field at $(5,2)$?

(a) $1/6$

(b) 0

(c) -1

(d) $5/6$

8. If $y' = y\cosh x$, then what is the slope of the direction field at $(5,0)$?

(a) 5

(b) -5

(c) 1

(d) 0

9. If $y' = (\sin x)(\cos x)$, then what is the slope of the direction field at $\left(\dfrac{\pi}{4}, \dfrac{\pi}{4}\right)$?

(a) 1

(b) $1/4$

(c) $1/2$

(d) $\dfrac{\sqrt{2}}{2}$

10. If $y' = 3\ln x - 2\ln y$, then what is the slope of the direction field at $(1,1)$?

(a) 0

(b) 2

(c) 1

(d) $2e$

11. Use Euler's method with a step-size of $\Delta x = 0.2$ to approximate the value of $y(1.6)$ if y satisfies the initial value problem, $y' = \sin(x-y)$, $y(1) = 1$.

(a) $y(1.6) \approx 1$

(b) $y(1.6) \approx 1.0397$

(c) $y(1.6) \approx 1.1102$

(d) $y(1.6) \approx 1.565$

12. Suppose that Euler's method is used to approximate the value of $y(t)$ if y satisfies the initial value problem, $y' = xe^y$, $y(2) = 0$. How big is each step-size, Δt, if 12 steps are used?

(a) $\Delta t = 2$

(b) $\Delta t = 1/6$

(c) $\Delta t = 1/12$

(d) $\Delta t = 1.2$

13. Consider the initial value problem $y' = y + t$, $y(0) = 1$. The exact solution of this problem is $y = -t - 1 + 2e^t$. If the Euler method with a step-size of $\Delta t = 0.25$ is used to approximate $y(1)$ then what is the absolute error of this approximation?

(a) 1.56

(b) 0.554

(c) 0.4464

(d) 0.138

Section 8.4

1. When using the method of integrating factors to solve the linear differential equation $(t+1)y' + ty = 5$, the correct integrating factor to multiply by is $\mu = e^{\int p(t)dt}$ where $p(t)$ is

(a) $p(t) = t$

(b) $p(t) = (t+1)$

(c) $p(t) = \frac{1}{t+1}$

(d) $p(t) = \frac{t}{t+1}$

2. When using the method of integrating factors to solve the linear differential equation $y' + y\sqrt{t} = g(t)$, the correct integrating factor to multiply by is

(a) $\mu = \sqrt{t}$

(b) $\mu = e^{\sqrt{t}}$

(c) $\mu = e^{(2/3)t^{3/2}}$

(d) $\mu = \frac{e^{t^{3/2}}}{\sqrt{t}}$

3. Solve the differential equation $\dfrac{dy}{dx} - 2y = 0$.

(a) $y = Ce^{2x}$

(b) $y = Ce^{-x}$

(c) $y = e^{Cx}$

(d) $y = e^{-Cx}$

4. Find the general solution of the differential equation $\frac{dy}{dt} - 3y = -2e^t$.

(a) $y = Ce^{-t}$

(b) $y = Ce^t$

(c) $y = e^t + Ce^{3t}$

(d) $y = -\ln|t| + C$

5. Find the unique solution of the initial-value problem $\{y' = 4y,\ y(1) = e^4\}$.

(a) $y = e^{4t}$

(b) $y = 4e^{4t}$

(c) $y = e^{-4t}$

(d) $y = -\frac{1}{t}$

6. Which of the following is the general solution of the differential equation $y' + 2y = e^{-2x}$?

(a) $y = Ce^{2x} - \frac{1}{4}e^{-2x}$

(b) $y = Ce^{-2x} + xe^{-2x}$

(c) $y = Ce^{2x} - \frac{1}{3}e^{-x}$

(d) $y = Ce^{-2x} + e^{-x}$

7. Find the unique solution of the initial-value problem $\{y' + 4y = t,\ y(0) = -2\}$.

(a) $y = -\frac{31}{16}e^{-4t} + \frac{1}{4}t - \frac{1}{16}$

(b) $y = \frac{33}{16}e^{-4t} + \frac{1}{4}t - \frac{1}{16}$

(c) $y = \frac{1}{16}e^{-4t} + \frac{1}{4}t - \frac{1}{16}$

(d) $y = -\frac{31}{16}e^{4t} - \frac{1}{4}t - \frac{1}{16}$

8. Which of the following is the general solution of the differential equation $ty' + 5y = t$?

(a) $y = \frac{C}{t^5} - \frac{t}{6}$

(b) $y = Ct^5 - \frac{t}{4}$

(c) $y = Ct^5 + \frac{t}{4}$

(d) $y = \frac{C}{t^5} + \frac{t}{6}$

9. Find the general solution of the differential equation $y' + y = \sin(x)$.

(a) $y = Ce^{-x} - \frac{1}{2}\cos(x) + \frac{1}{2}\sin(x)$

(b) $y = Ce^{-x} - \frac{1}{2}\cos(x) + \frac{1}{2}\sin(x)$

(c) $y = Ce^{x} - \frac{1}{2}\cos(x) - \frac{1}{2}\sin(x)$

(d) $y = Ce^{x} + \frac{1}{2}\cos(x) - \frac{1}{2}\sin(x)$

Chapter 8 Test

1. The order of the differential equation $4xy'' = 5y^3 + t$ is

(a) 0

(b) 1

(c) 2

(d) 3

2. Answer true or false. The differential equation $y' - 5y = 0$ is solved by $y = Ce^{5t}$.

3. Which of the following is the general solution of the differential equation $\frac{dy}{dt} - 9y = 0$?

(a) $y = Ce^{-3t}$

(b) $y = Ce^{9t}$

(c) $y = e^t + 9e^{9t} + C$

(d) $y = c_1e^{-3t} + c_2e^{3t}$

4. Which of the following is the general solution of the differential equation $y'' + 25y = 0$

(a) $y = c_1e^{-5t} + c_2e^{5t}$

(b) $y = c_1\cos(5t) + c_2\sin(5t)$

(c) $y = c_1e^{-5t} + c_2te^{-5t}$

(d) $y = c_1e^{5t} + c_2te^{5t}$

5. Answer true or false. The function $y = \cos(3t) + C$ solves the differential equation $y' + y = 0$.

6. Answer true or false. The function $y = Ce^t - 4$ solves the differential equation $y' - y = 4$.

7. If $y' = x\cos y$, then the slope of the direction field at the point $(5, 0)$ is

(a) 0

(b) -5

(c) 5

(d) 1

8. If $y' = 4x + 6y$ then the slope of the direction field at the point $(1, 2)$ is

(a) 14

(b) 16

(c) 10

(d) 3

9. If $y' = e^{xy}$ then the slope of the direction field at the point $(8,0)$ is

(a) 0

(b) 1

(c) 8

(d) e^8

10. If $y' = e^{3xy}$ then the slope of the direction field at the point $(7,0)$ is

(a) 0

(b) 1

(c) 7

(d) 21

11. Answer true or false. Suppose that a quantity $y = y(t)$ changes in such a way that $dy/dt = kt^{1/5}$, where $k > 0$. It can be said that the rate of change of the quantity is proportional to the fifth root of time.

12. Answer true or false. Suppose that a quantity $y = y(t)$ changes in such a way that $dy/dt = k\sqrt{t}$, where $k > 0$. It can be said that y increases at a rate that is proportional to the square root of time.

13. Suppose that an initial population of $20,000$ bacteria is growing at a rate of 4% per hour, and that $y = y(t)$ is the number of bacteria present after t hours. Write an expression for y.

(a) $y = 20,000 + 800t$

(b) $y = 20,000e^{0.04t}$

(c) $y = 20,000e^{1.04t}$

(d) $y = 20,000(1.04)^t$

14. Suppose that a radioactive substance decays with a half-life of 70 days. Find a formula that relates the amount present to t, if initially 60 g of the substance are present.

(a) $y = 60e^{-0.0099t}$

(b) $y = 60e^{-0.0116t}$

(c) $y = 60(0.5)^{-t/70}$

(d) $y = 60e^{-0.059t}$

15. If 480 g of a radioactive substance decays to 30 g in 17 years, then the half-life of the substance is

(a) 9.07 years

(b) 4.25 years

(c) 6.06 years

(d) 2.77 years

16. Answer true or false. The differential equation $y'' = 25t$ is solved by the function $y = c_1 \cos(5t) + c_2 \sin(5t)$.

17. If $y = y_o e^{kt}$ and $k < 0$ then the function being modeled is

(a) increasing.

(b) decreasing.

(c) constant.

(d) undetermined; more information is needed.

18. Answer true or false. An exponential decay model $y = y_o e^{kt}$ used to find the half-life of a substance always uses -0.2 for k.

19. Answer true or false. If $y(0) = 20$ and a substance grows at a rate of 13%, this situation may be modeled by $y = 20(0.13)^t$

20. Answer true or false. If $y(0) = y_o$ and a substance decreases at a rate of 8%, this situation may be modeled by $y = y_o(1 - 0.08t)$.

21. The unique solution to the differential equation $y' + ty = 8t$ with $y(0) = 2$ is

(a) $y = 8 - 6e^{-t^2/2}$

(b) $y = 4 - 2e^{-t^2}$

(c) $y = 8 - 6e^{-t/2}$

(d) $y = 8 - 6e^{t^2/2}$

Chapter 8: Answers to Sample Tests

Section 8.1

1. c	2. b	3. false	4. true	5. a	6. d	7. d	8. a
9. c	10. a						

Section 8.2

1. d	2. d	3. a	4. c	5. true	6. false	7. d	8. a
9. b	10. b	11. a	12. c	13. false	14. false		

Section 8.3

1. a	2. a	3. a	4. a	5. c	6. d	7. d	8. d
9. c	10. a	11. c	12. b	13. b			

Section 8.4

1. d	2. c	3. a	4. c	5. a	6. b	7. a	8. d
9. b							

Chapter 8 Test

1. c	2. true	3. b	4. b	5. false	6. true	7. c	8. b
9. b	10. b	11. true	12. true	13. d	14. a	15. b	16. false
17. d	18. false	19. false	20. false	21. a			