10.3 In Exercises 31–34, find the standard form of the equation of the hyperbola with the given characteristics. Text

31. Vertices: $(0, \pm 1)$; foci: $(0, \pm 2)$
32. Vertices: $(3, 3), (-3, 3)$; foci: $(4, 3), (-4, 3)$
33. Foci: $(0, 0), (8, 0)$; asymptotes: $y = \pm 2(x - 4)$
34. Foci: $(3, \pm 2)$; asymptotes: $y = \pm 2(x - 3)$

In Exercises 35–38, find the center, vertices, foci, and the equations of the asymptotes of the hyperbola, and sketch its graph using the asymptotes as an aid.

35. $\dfrac{(x-5)^2}{36} - \dfrac{(y+3)^2}{16} = 1$
36. $\dfrac{(y-1)^2}{4} - x^2 = 1$
37. $9x^2 - 16y^2 - 18x - 32y - 151 = 0$
38. $-4x^2 + 25y^2 - 8x + 150y + 121 = 0$

39. **LORAN** Radio transmitting station A is located 200 miles east of transmitting station B. A ship is in an area to the north and 40 miles west of station A. Synchronized radio pulses transmitted at 186,000 miles per second by the two stations are received 0.0005 second sooner from station A than from station B. How far north is the ship?

40. **LOCATING AN EXPLOSION** Two of your friends live 4 miles apart and on the same "east-west" street, and you live halfway between them. You are having a three-way phone conversation when you hear an explosion. Six seconds later, your friend to the east hears the explosion, and your friend to the west hears it 8 seconds after you do. Find equations of two hyperbolas that would locate the explosion. (Assume that the coordinate system is measured in feet and that sound travels at 1100 feet per second.)

In Exercises 41–44, classify the graph of the equation as a circle, a parabola, an ellipse, or a hyperbola.

41. $5x^2 - 2y^2 + 10x - 4y + 17 = 0$
42. $-4y^2 + 5x + 3y + 7 = 0$
43. $3x^2 + 2y^2 - 12x + 12y + 29 = 0$
44. $4x^2 + 4y^2 - 4x + 8y - 11 = 0$

10.5 In Exercises 45–48, rotate the axes to eliminate the xy-term in the equation. Then write the equation in standard form. Sketch the graph of the resulting equation, showing both sets of axes.

45. $xy + 3 = 0$
46. $x^2 - 4xy + y^2 + 9 = 0$
47. $5x^2 - 2xy + 5y^2 - 12 = 0$
48. $4x^2 + 8xy + 4y^2 + 7\sqrt{2}x + 9\sqrt{2}y = 0$

In Exercises 49–52, (a) use the discriminant to classify the graph, (b) use the Quadratic Formula to solve for y, and (c) use a graphing utility to graph the equation.

49. $16x^2 - 24xy + 9y^2 - 30x - 40y = 0$
50. $13x^2 - 8xy + 7y^2 - 45 = 0$
51. $x^2 + y^2 + 2xy + 2\sqrt{2}x - 2\sqrt{2}y + 2 = 0$
52. $x^2 - 10xy + y^2 + 1 = 0$

10.6 In Exercises 53 and 54, (a) create a table of x- and y-values for the parametric equations using $t = -2, -1, 0, 1$, and 2, and (b) plot the points (x, y) generated in part (a) and sketch a graph of the parametric equations.

53. $x = 3t - 2$ and $y = 7 - 4t$
54. $x = \dfrac{1}{4}t$ and $y = \dfrac{6}{t+3}$

In Exercises 55–60, (a) sketch the curve represented by the parametric equations (indicate the orientation of the curve) and (b) eliminate the parameter and write the corresponding rectangular equation whose graph represents the curve. Adjust the domain of the resulting rectangular equation, if necessary. (c) Verify your result with a graphing utility.

55. $x = 2t$, $y = 4t$
56. $x = 1 + 4t$, $y = 2 - 3t$
57. $x = t^2$, $y = \sqrt{t}$
58. $x = t + 4$, $y = t^2$
59. $x = 3\cos\theta$, $y = 3\sin\theta$
60. $x = 3 + 3\cos\theta$, $y = 2 + 5\sin\theta$

61. Find a parametric representation of the line that passes through the points $(-4, 4)$ and $(9, -10)$.
62. Find a parametric representation of the circle with center $(5, 4)$ and radius 6.
63. Find a parametric representation of the ellipse with center $(-3, 4)$, major axis horizontal and eight units in length, and minor axis six units in length.
64. Find a parametric representation of the hyperbola with vertices $(0, \pm 4)$ and foci $(0, \pm 5)$.

10.7 In Exercises 65–68, plot the point given in polar coordinates and find two additional polar representations of the point, using $-2\pi < \theta < 2\pi$.

65. $\left(2, \dfrac{\pi}{4}\right)$
66. $\left(-5, -\dfrac{\pi}{3}\right)$
67. $(-7, 4.19)$
68. $(\sqrt{3}, 2.62)$

In Exercises 69–72, a point in polar coordinates is given. Convert the point to rectangular coordinates.

69. $\left(-1, \frac{\pi}{3}\right)$ **70.** $\left(2, \frac{5\pi}{4}\right)$

71. $\left(3, \frac{3\pi}{4}\right)$ **72.** $\left(0, \frac{\pi}{2}\right)$

In Exercises 73–76, a point in rectangular coordinates is given. Convert the point to polar coordinates.

73. $(0, 1)$ **74.** $(-\sqrt{5}, \sqrt{5})$

75. $(4, 6)$ **76.** $(3, -4)$

In Exercises 77–82, convert the rectangular equation to polar form.

77. $x^2 + y^2 = 81$ **78.** $x^2 + y^2 = 48$

79. $x^2 + y^2 - 6y = 0$ **80.** $x^2 + y^2 - 4x = 0$

81. $xy = 5$ **82.** $xy = -2$

In Exercises 83–88, convert the polar equation to rectangular form.

83. $r = 5$ **84.** $r = 12$

85. $r = 3\cos\theta$ **86.** $r = 8\sin\theta$

87. $r^2 = \sin\theta$ **88.** $r^2 = 4\cos 2\theta$

In Exercises 89–98, determine the symmetry of r, the maximum value of $|r|$, and any zeros of r. Then sketch the graph of the polar equation (plot additional points if necessary).

89. $r = 6$ **90.** $r = 11$

91. $r = 4\sin 2\theta$ **92.** $r = \cos 5\theta$

93. $r = -2(1 + \cos\theta)$ **94.** $r = 1 - 4\cos\theta$

95. $r = 2 + 6\sin\theta$ **96.** $r = 5 - 5\cos\theta$

97. $r = -3\cos 2\theta$ **98.** $r^2 = \cos 2\theta$

In Exercises 99–102, identify the type of polar graph ~~and use a graphing utility to graph the equation~~.

use our process

99. $r = 3(2 - \cos\theta)$ **100.** $r = 5(1 - 2\cos\theta)$

101. $r = 8\cos 3\theta$ **102.** $r^2 = 2\sin 2\theta$

In Exercises 103–106, identify the conic and sketch its graph.

use our process

103. $r = \dfrac{1}{1 + 2\sin\theta}$ **104.** $r = \dfrac{6}{1 + \sin\theta}$

105. $r = \dfrac{4}{5 - 3\cos\theta}$ **106.** $r = \dfrac{16}{4 + 5\cos\theta}$

In Exercises 107–110, find a polar equation of the conic with its focus at the pole.

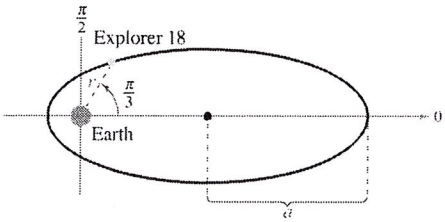
107. Parabola Vertex: $(2, \pi)$

108. Parabola Vertex: $(2, \pi/2)$

109. Ellipse Vertices: $(5, 0), (1, \pi)$

110. Hyperbola Vertices: $(1, 0), (7, 0)$

111. EXPLORER 18 On November 27, 1963, the United States launched Explorer 18. Its low and high points above the surface of Earth were 119 miles and 122,800 miles, respectively. The center of Earth was at one focus of the orbit (see figure). Find the polar equation of the orbit and find the distance between the surface of Earth (assume Earth has a radius of 4000 miles) and the satellite when $\theta = \pi/3$.



112. ASTEROID An asteroid takes a parabolic path with Earth as its focus. It is about 6,000,000 miles from Earth at its closest approach. Write the polar equation of the path of the asteroid with its vertex at $\theta = \pi/2$. Find the distance between the asteroid and Earth when $\theta = -\pi/3$.

EXPLORATION

TRUE OR FALSE? In Exercises 113–115, determine whether the statement is true or false. Justify your answer.

113. The graph of $\frac{1}{4}x^2 - y^4 = 1$ is a hyperbola.

114. Only one set of parametric equations can represent the line $y = 3 - 2x$.

115. There is a unique polar coordinate representation of each point in the plane.

116. Consider an ellipse with the major axis horizontal and 10 units in length. The number b in the standard form of the equation of the ellipse must be less than what real number? Explain the change in the shape of the ellipse as b approaches this number.

117. What is the relationship between the graphs of the rectangular and polar equations?

(a) $x^2 + y^2 = 25, \quad r = 5$

(b) $x - y = 0, \quad \theta = \dfrac{\pi}{4}$