

67. A psychologist conducts an experiment to determine the effect of sleep loss on job performance. Let $p = f(t)$ be the number of minutes it takes the average person to complete a particular task if he or she has lost t minutes of sleep, where $t = 0$ represents exactly 8 hours of sleep. For instance, $f(60)$ is the amount of time it takes the average person to complete the task after sleeping for only 7 hours. Let $p_0 = f(0)$ and let t_1 , t_2 , and t_3 be positive constants. Explain what the following statements tell you about sleep loss and job performance.

- (a) $f(30) = p_0 + 5$
- (b) $f(t_1) = 2p_0$
- (c) $f(2t_1) = 1.5f(t_1)$
- (d) $f(t_2 + 60) = f(t_2 + 30) + 10$

68. Give a formula for a function whose domain is all non-negative values of x except $x = 3$.

69. Give a formula for a function that is undefined for $x = 8$ and for $x < 4$, but is defined everywhere else.

70. Many printing presses are designed with large plates that print a fixed number of pages as a unit. Each unit is called a signature. A particular press prints signatures of 16 pages each. Suppose $C(p)$ is the cost of printing a book of p pages, assuming each signature printed costs \$0.14.

- (a) What is the cost of printing a book of 128 pages? 129 pages? p pages?
- (b) What are the domain and range of C ?
- (c) Graph $C(p)$ for $0 \leq p \leq 128$.

71. Table 2.25 shows the population, P , in millions, of Ireland¹⁶ at various times between 1780 and 1910, with t in

years since 1780.

- (a) When was the population increasing? Decreasing?
- (b) For each successive time interval, construct a table showing the average rate of change of the population.
- (c) From the table you constructed in part (b), when is the graph of the population concave up? Concave down?
- (d) When was the average rate of change of the population the greatest? The least? How is this related to part (c)? What does this mean in human terms?
- (e) Graph the data in Table 2.25 and join the points by a curve to show the trend in the data. From this graph identify where the curve is increasing, decreasing, concave up and concave down. Compare your answers to those you got in parts (a) and (c). Identify the region you found in part (d).
- (f) Something catastrophic happened in Ireland between 1780 and 1910. When? What happened in Ireland at that time to cause this catastrophe?

Table 2.25 The population of Ireland from 1780 to 1910, where $t = 0$ corresponds to 1780

t	0	20	40	60	70	90	110	130
P	4.0	5.2	6.7	8.3	6.9	5.4	4.7	4.4

CHECK YOUR UNDERSTANDING

Are the statements in Problems 1–42 true or false? Give an explanation for your answer.

1. If $f(t) = 3t^2 - 4$ then $f(2) = 0$.
2. If $f(x) = x^2 - 9x + 10$ then $f(b) = b^2 - 9b + 10$.
3. If $f(x) = x^2$ then $f(x + h) = x^2 + h^2$.
4. If $q = \frac{1}{\sqrt{z^2 + 5}}$ then the values of z that make $q = \frac{1}{3}$ are $z = \pm 2$.
5. If $W = \frac{t + 4}{t - 4}$ then when $t = 8$, $W = 1$.
6. If $f(t) = t^2 + 64$ then $f(0) = 64$.
7. If $f(x) = 0$ then $x = 0$.
8. If $f(x) = x^2 + 2x + 7$ then $f(-x) = f(x)$.

9. If $g(x) = \frac{3}{\sqrt{x^2 + 4}}$ then $g(x)$ can never be zero.

10. If $h(p) = -6p + 9$ then $h(3) + h(4) = h(7)$.

11. The domain of a function is the set of input values.

12. If a function is being used to model a real-world situation, the domain and range are often determined by the constraints of the situation being modeled.

13. The domain of $f(x) = \frac{4}{x - 3}$ consists of all real numbers x , $x \neq 0$.

14. If $f(x) = \sqrt{2 - x}$, the domain of f consists of all real numbers $x \geq 2$.

15. The range of $f(x) = \frac{1}{x}$ is all real numbers.

¹⁶Adapted from D. N. Burghes and A. D. Wood, *Mathematical Models in the Social, Management and Life Science*, p. 104 (Ellis Horwood, 1980).

16. The range of $y = 4 - \frac{1}{x}$ is $0 < y < 4$.
17. If $f(x) = \frac{2}{5}x + 6$ and its domain is $15 \leq x \leq 20$ then the range of f is $12 \leq x \leq 14$.
18. The domain of $f(x) = \frac{x}{\sqrt{x^2+1}}$ is all real numbers.
19. The graph of the absolute value function $y = |x|$ has a V shape.
20. The domain of $f(x) = |x|$ is all real numbers.
21. If $f(x) = |x|$ and $g(x) = |-x|$ then for all x , $f(x) = g(x)$.
22. If $f(x) = |x|$ and $g(x) = -|x|$ then for all x , $f(x) = g(x)$.
23. If $y = \frac{x}{|x|}$ then $y = 1$ for $x \neq 0$.
24. If $f(x) = \begin{cases} 3 & \text{if } x < 0 \\ x^2 & \text{if } 0 \leq x \leq 4, \\ 7 & \text{if } x > 4 \end{cases}$ then $f(3) = 0$.
25. Let $f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } 0 \leq x \leq 4. \\ -x & \text{if } x > 4 \end{cases}$ If $f(x) = 4$ then $x = 2$.
26. If $f(3) = 5$ and f is invertible, then $f^{-1}(3) = 1/5$.
27. If $h(7) = 4$ and h is invertible, then $h^{-1}(4) = 7$.
28. If $f(x) = \frac{3}{4}x - 6$ then $f^{-1}(8) = 0$.
29. If $R = f(S) = \frac{2}{3}S + 8$ then $S = f^{-1}(R) = \frac{3}{2}(R - 8)$.
30. In general $f^{-1}(x) = (f(x))^{-1}$.
31. If $f(x) = \frac{x}{x+1}$ then $f(t^{-1}) = \frac{1/t}{1/t+1}$.
32. The units of the output of a function are the same as the units of output of its inverse.
33. The functions $f(x) = 2x + 1$ and $g(x) = \frac{1}{2}x - 1$ are inverses.
34. If $q = f(x)$ is the quantity of rice in tons required to feed x million people for a year and $p = g(q)$ is the cost, in dollars, of q tons of rice, then $g(f(x))$ is the dollar cost of feeding x million people for a year.
35. If $f(t) = t+2$ and $g(t) = 3t$, then $g(f(t)) = 3(t+2) = 3t + 6$.
36. A fireball has radius $r = f(t)$ meters t seconds after an explosion. The volume of the ball is $V = g(r)$ meter³ when it has radius r meters. Then the units of measurement of $g(f(t))$ are meter³/sec.
37. If the graph of a function is concave up, then the average rate of change of a function over an interval of length 1 increases as the interval moves from left to right.
38. The function f in the table could be concave up.

x	-2	0	2	4
$f(x)$	5	6	8	12

39. The function g in the table could be concave down.

t	-1	1	3	5
$g(t)$	9	8	6	3

40. A straight line is concave up.
41. A function can be both decreasing and concave down.
42. If a function is concave up, it must be increasing.