

## Inverse Trig and Solving Trig Equations with a Calculator

**Concept Check** Tell whether each statement is true or false. If false, tell why.

- The ranges of the inverse sine and inverse cosine functions are the same.
- The ranges of the inverse tangent and inverse cotangent functions are the same.
- It is true that  $\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2}$ , and therefore  $\arcsin\left(-\frac{1}{2}\right) = \frac{11\pi}{6}$ .
- For all  $x$ ,  $\tan(\tan^{-1}x) = x$ .

Give the exact real number value of  $y$ . Do not use a calculator.

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| 5. $y = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ | 6. $y = \arccos\left(-\frac{1}{2}\right)$                       | 7. $y = \tan^{-1}(-\sqrt{3})$                    |
| 8. $y = \arcsin(-1)$                              | 9. $y = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$              | 10. $y = \arctan\left(\frac{\sqrt{3}}{3}\right)$ |
| 11. $y = \sec^{-1}(-2)$                           | 12. $y = \operatorname{arccsc}\left(\frac{2\sqrt{3}}{3}\right)$ | 13. $y = \operatorname{arccot}(-1)$              |

Give the degree measure of  $\theta$ . Do not use a calculator.

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| 14. $\theta = \arccos\left(\frac{1}{2}\right)$ | 15. $\theta = \arcsin\left(-\frac{\sqrt{3}}{2}\right)$ | 16. $\theta = \tan^{-1} 0$ |
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Use a calculator to give the degree measure of  $\theta$ .

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|------------------------------------|--|--|
| 17. $\theta = \arctan 1.7804675$   | 18. $\theta = \sin^{-1}(-.66045320)$           | 19. $\theta = \cos^{-1}.80396577$                |
| 20. $\theta = \cot^{-1} 4.5046388$ | 21. $\theta = \operatorname{arcsec} 3.4723155$ | 22. $\theta = \operatorname{csc}^{-1} 7.4890096$ |
23. Explain why  $\sin^{-1} 3$  cannot be defined.
24.  $\operatorname{Arcsin}(\sin 5\pi/6) \neq 5\pi/6$ . Explain why.
25. What is the domain of the arccotangent function?
26. What is the range of the arcsecant function as defined in this text?

Evaluate the following without using a calculator.

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| 27. $\sin\left(\sin^{-1}\frac{1}{2}\right)$  | 28. $\tan\left(\tan^{-1}\frac{2}{3}\right)$                | 29. $\cos(\arccos(-1))$  | 30. $\sin\left(\arcsin\left(-\frac{\sqrt{3}}{2}\right)\right)$ |
| 31. $\arccos\left(\cos\frac{3\pi}{4}\right)$ | 32. $\operatorname{arcsec}(\sec \pi)$                      | 33. $\tan^{-1}\left(\tan\frac{\pi}{4}\right)$                  |  |
| 34. $\cos^{-1}(\cos 0)$                      | 35. $\sin\left(\arccos\frac{3}{4}\right)$                  | 36. $\cos(\arctan 3)$  |  |
| 37. $\cos(\operatorname{csc}^{-1}(-2))$      | 38. $\sec\left(2\sin^{-1}\left(-\frac{1}{3}\right)\right)$ | 39. $\tan\left(\arcsin\frac{3}{5} + \arccos\frac{5}{7}\right)$ |  |

Write each of the following as a non-trigonometric expression in  $u$ .

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|-------------------------|--|--|
| 40. $\sin(\tan^{-1} u)$ | 41. $\cos\left(\arctan\frac{u}{\sqrt{1-u^2}}\right)$ | 42. $\tan\left(\operatorname{arcsec}\frac{\sqrt{u^2+1}}{u}\right)$ |
|-------------------------|--|--|

Graph each of the following, and give the domain and range.

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|-----------------------|-----------------------|-----------------------------------|
| 43. $y = \sin^{-1} x$ | 44. $y = \cos^{-1} x$ | 45. $y = \operatorname{arccot} x$ |
|-----------------------|-----------------------|-----------------------------------|

Solve each equation for solutions in the interval  $[0, 2\pi)$ . Use a calculator in Exercises 47 and 48.

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|---|----------------------------|-------------------------------------|
| 46. $\sin^2 x = 1$                      | 47. $2 \tan x - 1 = 0$     | 48. $3 \sin^2 x - 5 \sin x + 2 = 0$ |
| 49. $\tan x = \cot x$                   | 50. $\sec^2 2x = 4$        | 51. $\tan^2 2x - 1 = 0$             |
| 52. $\sec\frac{x}{2} = \cos\frac{x}{2}$ | 53. $\cos 2x + \cos x = 0$ | 54. $4 \sin x \cos x = \sqrt{3}$    |

Solve each equation for solutions in the interval  $[0^\circ, 360^\circ)$ . When appropriate, use a calculator and express solutions to the nearest tenth of a degree.

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|---|--|---|
| 55. $\sin^2 \theta + 3 \sin \theta + 2 = 0$ | 56. $2 \tan^2 \theta = \tan \theta + 1$          | 57. $\sin 2\theta = \cos 2\theta + 1$       |
| 58. $2 \sin 2\theta = 1$                    | 59. $3 \cos^2 \theta + 2 \cos \theta - 1 = 0$    | 60. $5 \cot^2 \theta - \cot \theta - 2 = 0$ |
| 61. $\sin 2\theta + \sin 4\theta = 0$       | 62. $\cos \theta - \cos 2\theta = 2 \cos \theta$ |   |