CONTINUITY AP CALCULUS

- A polynomial is continuous everywhere.
- A rational function is continuous at every point where the denominator is nonzero and has discontinuities at the points where the denominator is zero.

POINT CONTINUITY

$f(x)$ is continuous at $x = c$ provided that

1. $f(c)$ is defined.
2. $\lim_{x \to c} f(x)$ exists.
3. $\lim_{x \to c} f(x) = f(c)$

REMOVABLE DISCONTINUITY

A function f is said to have a removable discontinuity at $x = c$ provided that

1. $\lim_{x \to c} f(x)$ exists at $x = c$ and
2. f is not continuous at $x = c$, either because
   a) f is not defined at $x = c$
   b) $f(c) \neq \lim_{x \to c} f(x)$

INTERVAL CONTINUITY

$f(x)$ is continuous on a closed interval $[a, b]$ provided that

1. f is continuous on $(a. b)$.
2. f is continuous from the right **at** $x = a$.
3. f is continuous from the left **at** $x = b$.

INTERMEDIATE VALUE THEOREM

If f is continuous on a closed interval $[a, b]$ and k is any number between $f(a)$ and $f(b)$, inclusive, then there is at least one number x in the interval $[a, b]$ such that $f(x) = k$.

If f is continuous on a closed interval $[a, b]$, and if $f(a)$ and $f(b)$are nonzero and have opposite signs, then there is at least one solution of the equation $f(x) = 0$ in the interval (a, b).

PROPERTIES

If the functions f and g are continuous at $x = c$, then

1. $f \pm g$ is continuous at $x = c$.
2. $f \cdot g$ is continuous at $x = c$.
3. f/g is continuous at $x = c$ if $g(c) \neq 0$.

COMPOSITE FUNCTIONS

If $\lim_{x \to c} g(x) = L$ and if the function f is continuous at L, then $\lim_{x \to c} f(g(x)) = f\left(\lim_{x \to c} g(x)\right)$

Squeeze Theorem (See AP Video 1.8)

If $g(x) \leq f(x) \leq h(x)$ for all values of x in an open interval containing c, except possibly at c itself, and if $\lim_{x\to c} g(x) = L = \lim_{x\to c} h(x)$ then the $\lim_{x\to c} f(x)$ exists and equals L.

Trig Limits to Memorize

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{\sin bx}{bx} = 1$$

$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$$

Example:

$$\lim_{x\to 0} \frac{\sin 5x}{4x} =$$