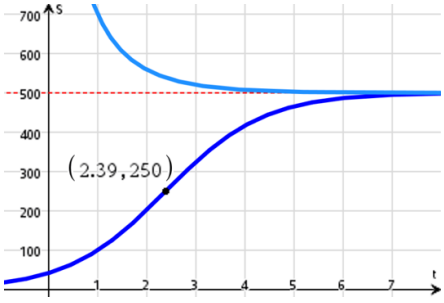
Calculus BC - 2022 AP Live Review Session 2: Differential Equations

What Do We Need to Know ? (Slope Fields, Differential Equations, Euler's Method, Logistic Growth)

Law of Natural Growth/Decay	$\frac{dy}{dt} = ky$
Newton's Law of Cooling *	$\frac{dT_O}{dt} = k\left(T_O - T_E\right)$
* Not explicitly tested on the AP Calculus exam	



Logistic differential equation	$\frac{dy}{dy} = ky\left(1 - \frac{y}{L}\right)$ or $\frac{dy}{dt} = ky\left(L - y\right)$
End behavior and carrying capacity	$\lim_{t\to\infty} y = L$
Fastest growth rate	occurs when $y = \frac{L}{2}$
Concave up: y increasing at an increasing rate	$\frac{d^2y}{dt^2} > 0$ when $y < \frac{L}{2}$
Concave down: y increasing at a decreasing rate	$\frac{d^2y}{dt^2} < 0$ when $\frac{L}{2} < y < L$
Increasing behavior (if initial amount < L)	$\frac{dy}{dt} > 0$ for all t

Topic Name	Topic #
Slope Fields	7.3-7.4
Euler's Method	7.5
Solving a Differential Equation	7.6-7.7
Exponential Models with Differential Equations	7.8
Logistic Models with Differential Equations	7.9

Multiple Choice Practice

1. Level: AP2

Which of the following is a logistic differential equation?

(A) $\frac{dy}{dt} = 2y\left(1+\frac{y}{8}\right)$

(B) $\frac{dR}{dt} = 3\left(1-\frac{R}{300}\right)$

(C) $\frac{dQ}{dt} = 4Q\left(1-\frac{Q}{175}\right)$

(D) $\frac{dw}{dt} = 5w\left(1-\frac{w^2}{15}\right)$

2. Level: AP2

Which of the following differential equations is *not* separable?

(A) $\frac{dy}{dx} = 3yx - x$

(B) $\frac{dy}{dx} = e^{x-2y}$

(C) $\frac{dy}{dx} = 3y - x$

(D) $\frac{dy}{dx} = \frac{\sin(2x)+3}{y-1}$

3. Level: AP2

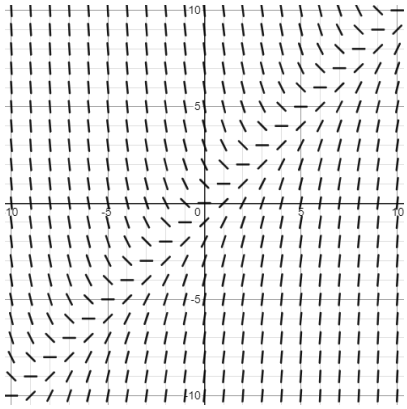
Consider the differential equation $\frac{dy}{dx} = x + 2y$ where $y = f(x)$ be the particular solution through the point $(2,3)$. Starting at $x = 2$, which of the following gives the correct set-up for the first step of Euler's method when approximating $f(1)$ with two steps of equal size?

(A) $y_1 = 3+\frac{1}{2}(8)$

(B) $y_1 = 3-\frac{1}{2}(8)$

(C) $y_1 = 3-1(8)$

(D) $y_1 = 3+1(8)$



4. Level: AP3

Which of the following differential equations models the slope field shown above?

(A) $\frac{dy}{dx} = y - x$

(B) $\frac{dy}{dx} = x - y$

(C) $\frac{dy}{dx} = x + y$

(D) $\frac{dy}{dx} = x^2 - y$

5. Level: AP4

A hilarious meme about Bryan and Tony is spreading among AP Calculus students. The number of students that have seen the meme follows a logistic curve modeled by the function $S(t)$ where t is the time in days. The number of students that have seen the meme is increasing at a rate of 125 students per day when S reaches half the carrying capacity of 5000 students.

Which of the following is the logistic differential equation for $\frac{dS}{dt}$?

(A) $\frac{dS}{dt} = \frac{1}{10} S\left(1 - \frac{S}{5000}\right)$

(B) $\frac{dS}{dt} = \frac{1}{125} S\left(1 - \frac{S}{2500}\right)$

(C) $\frac{dS}{dt} = 10S\left(1 - \frac{S}{2500}\right)$

(D) $\frac{dS}{dt} = 125S\left(1 - \frac{S}{5000}\right)$

6. Level: AP4

Given $x^2 dy = y\,dx$, which of the following could be $y = f(x)$?

(A) $y = \sqrt{\frac{2}{3}x^3 + 6}$

(B) $y = e^{\frac{-1}{x}} + 5$

(C) $y = 2e^{\frac{-1}{x}}$

(D) $y = \frac{2}{3}x^2$

7. Level: AP4

The function $y = e^{2x} + \cos(x)$ is a solution for which of the following differential equations?

(A) $y'' - y + 2\cos(x) = 0$

(B) $y'' - 4y + 3\cos(x) = 0$

(C) $4y'' - y + 5\cos(x) = 0$

(D) $y'' - 4y + 5\cos(x) = 0$

8. Level: AP4

A population of meerkats grows according to the logistic differential equation

$$\frac{dM}{dt} = 3M - 0.01M^2$$

with M representing the number of meerkats at time *t* in years. Which of the following statements are true?

I. If $M > 300$, the population of meerkats is decreasing.
II. The rate at which the meerkat population is growing is greatest at $M = 150$.
III. $\lim_{t\to\infty} M(t) = 300$

(A) I and II only

(B) II and III only

(C) I and III only

(D) I, II and III

x	3	3.25	3.5	3.75	4
$f'(x)$	4.5	5.5	5	4	6

9. Level: AP5

Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = f'(x)$ passing through the point $(4,1)$. Selected values of $f'(x)$ are given in the table above. Using Euler's method, starting at $x = 4$ with two steps of equal size, what is the approximation for $f(3)$?

(A) -7.5

(B) -4.5

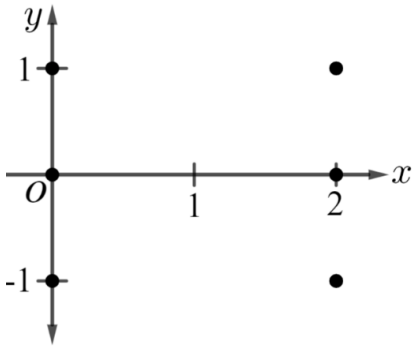
(C) -1.5

(D) 6.5

Free Response Practice

1. Consider the differential equation $\frac{dy}{dx} = x^2 - 4y$. Let $y = f(x)$ be the solution to the differential equation with the initial condition $f(2) = 1$.

(a) On the axes provided, sketch a slope field for the given differential equation at the 6 points indicated.



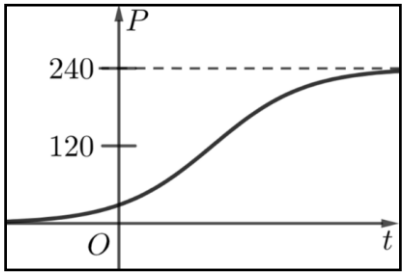
(b) Let $k(x) = f(x) - 3x + 6$. Does k have a local minimum, local maximum, or neither at $x = 2$? Justify your answer.

(c) Find the value of $\lim_{x \to 4} \dfrac{3e^{12-2x} - \int_1^x te^t dt}{f\left(\frac{x}{2}\right) - 2x + 7}$ or state that it does not exist. Justify your answer.

(d) Let $y = f(x)$ be the particular solution to the given differential with $f(2) = 1$. Use Euler's method, starting at $x = 2$ with 2 steps of equal size, to approximate $f(1)$.

Additional Multiple-Choice Practice

10. Level: AP3



Consider the logistic differential equation $\frac{dP}{dt}$ where the graph of $y = P(t)$ is shown above. Which of the following could be an equation for $\frac{dP}{dt}$?

(A) $\frac{dP}{dt} = 120P$

(B) $\frac{dP}{dt} = 2P\left(1 - \frac{P}{120}\right)$

(C) $\frac{dP}{dt} = 0.1P\left(240 - P\right)$

(D) $\frac{dP}{dt} = 0.5P\left(P - 240\right)$

11. Level: AP3

Find the particular solution to the differential equation $\frac{dy}{dx} = \frac{3x^2}{2y}$, where $y(2) = 3$.

(A) $y = \sqrt{x^3 + 1}$

(B) $y = 3e^{\frac{x^3}{2} - 4}$

(C) $y = \frac{3}{2}x^2 - 3$

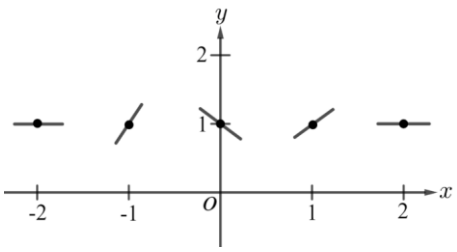
(D) $y = x^3 + 8$

12. Level: AP4

Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = \frac{2x-6}{y}$ with initial condition $(1,-2)$. What is the approximation for $f(2)$ obtained by using Euler's method, with two steps, of equal size, starting at $x = 1$?

(A) -8 (B) -1 (C) 0 (D) $\frac{1}{2}$

13. Level: AP4



Let $g(x) = f(2x - x^2)$. The slope field for $\frac{dy}{dx}$, the derivative of $f(x)$, is shown above. Which of the following could be $g'(2)$?

(A) -2

(B) -1

(C) 0

(D) 2

14. Level: AP5

Solve the differential equation $\frac{dy}{dx} = \frac{1}{(x+1)(x+2)}$, where $y(0) = 3 - \ln 2$.

(A) $y = \ln|x+1| - \ln|x+2| + 3$

(B) $y = \ln\left|\frac{x+2}{x+1}\right| + 3 - \ln 2$

(C) $y = \ln\left|\frac{x+1}{2(x+2)}\right| + 3$

(D) $y = \ln|x+2| - \ln|x+1| + 3$

15. Level: AP5

Consider the logistic differential equation $\frac{dy}{dx} = 2y\left(1-\frac{y}{50}\right)$ with the condition $y(7) = 100$. At $x = 10$, which of the following statements is correct?

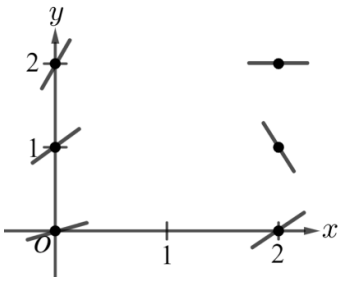
(A) y is increasing at an increasing rate

(B) y is increasing at a decreasing rate

(C) y is decreasing and concave up

(D) y is decreasing and concave down

16. Level: AP4



Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx}$ where $f(2) = 1$. A slope field for the $\frac{dy}{dx}$ is shown above. Which of the following could be the equation of the line tangent to $f(x)$?

(A) $y = \frac{1}{2}(x-2)+1$

(B) $y = -2(x-2)+1$

(C) $y = 3(x-2)+1$

(D) $y = 1$

17. Level: AP3

Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 3y\left(1-\frac{y}{80}\right)$ through the point $(7, 20)$. What is the value of $\lim_{x\to\infty} f(x)$?

(A) 40

(B) 45

(C) 80

(D) ∞