

## 5.6 Fundamental Theorem of Calculus

In Exercises 1-4, find formulas for the functions represented by the definite integrals.

1.  $\int_2^x (12t^2 - 8t) dt$

3.  $\int_1^{x^2} t dt$

2.  $\int_{-\pi/4}^x \sec^2 \theta d\theta$

4.  $\int_{3x}^{9x+2} e^{-u} du$

5.  $F(x) = \int_1^x t^3 + 1 dt$

- Find  $F(3)$  and explain geometrically what you found.
- Find  $F'(2)$ . What does the sign of  $F'(2)$  tell about  $F(x)$ ?
- Find  $F''(-2)$ . What does the sign of  $F''(-2)$  tell about  $F(x)$ ?

**5.6 2<sup>nd</sup> Fundamental Theorem of Calculus**  $\frac{d}{dx} \int_c^{g(x)} f(t) dt = f(g(x)) \cdot g'(x)$

In Exercises 6-9, calculate the derivative.

6.  $\frac{d}{dx} \int_0^{x^2} \frac{t dt}{t+1}$

7.  $\frac{d}{dx} \int_1^{1/x} \cos^3 t dt$

8.  $\frac{d}{ds} \int_{-6}^{\cos s} u^4 du$

9.  $\frac{d}{dx} \int_1^{x^3} \frac{\sin t}{t} dt$

10. Find  $G(1)$ ,  $G(0)$ , and  $G(\pi/4)$ , where  $G(x) = \int_1^x \tan t dt$ .

11. Find  $H(-2)$  and  $H'(-2)$ , where  $H(x) = \int_{-2}^x \frac{du}{u^2 + 1}$ .

12. Let  $F(x) = \int_2^x \frac{t+3}{t-1} dt$ . Find  $F(2)$ ,  $F'(2)$ , and  $F''(2)$ .

13. Let  $G(x) = \int_1^x (t^2 - 2) dt$ . Calculate  $G(1)$ ,  $G'(1)$  and  $G'(2)$ . Then find the equation for  $G(x)$ . (solve the initial value problem)

14.  $F(x) = \int_2^x 4t^2 + 4 dt$ .

- Evaluate  $F(2)$ ,  $F'(2)$ , and  $F''(2)$ .

## 5.6 Applications of 2<sup>nd</sup> Fundamental Theorem of Calculus $\frac{d}{dx} \int_c^{f(x)} f(t) dt$

15.  $F(x) = \int_2^x \frac{t-2}{t+1} dt$  **Show all work.**

- Find the intervals where  $F(x)$  is increasing and decreasing.
- Find all critical points and stationary points for  $F(x)$ .
- Find the concavity of  $F(x)$ .
- Find all the inflection points of  $F(x)$ .
- Include a sketch of  $F(x)$  for **extra credit**

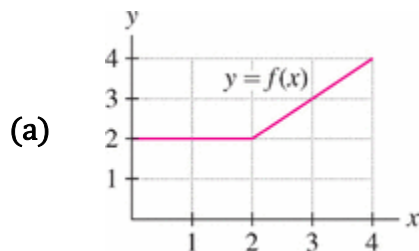
16. For the given function,  $F(x) = \int_2^x t^2 + t - 6 dt$

- Find  $F(0)$
- Find  $F'(2)$
- Find the intervals of concavity.
- Identify the location of any inflection points.

17.  $F(x) = \int_2^x 4t^2 + 4 dt$ .

- Evaluate  $F(2)$ ,  $F'(2)$ , and  $F''(2)$ .
- Find all intervals of increase and decrease
- Find all intervals of concavity
- Identify any stationary points, relative max/mins, and inflection points.

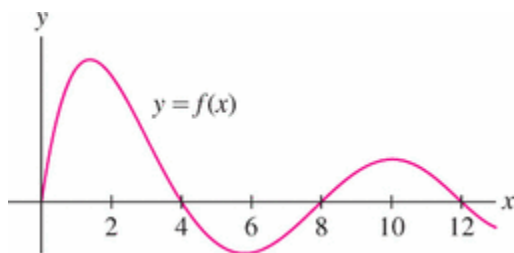
18. Let  $A(x) = \int_0^x f(t) dt$  for  $f(x)$  in **Figure 8**.



**FIGURE 8**

Calculate  $A(2)$ ,  $A(3)$ ,  $A'(2)$ , and  $A'(3)$ .

19. Let  $A(x) = \int_0^x f(t) dt$ , with  $f(x)$  given. Determine



- The intervals on which  $A(x)$  is increasing and decreasing
- The values  $x$  where  $A(x)$  has a local min or max
- The inflection points of  $A(x)$
- The intervals where  $A(x)$  is concave up or concave down