6.6 Fundamental Theorem of Calculus

In Exercises 1-4, find formulas for the functions represented by the definite integrals.

1. $\int_2^x (12t^2 - 8t)\,dt$ $4x^3 - 4x^2 - 16$

2. $\int_{-\pi/4}^{x} \sec^2\theta\,d\theta$ $\tan x + 1$

3. $\int_1^{x^2} t\,dt$ $\frac{x^4}{2} - \frac{1}{2}$

4. $\int_{3x}^{9x+2} e^{-u}\,du$ $-e^{-9x-2} + e^{-3x}$

5. $F(x) = \int_1^x t^3 + 1\,dt$
   a. Find $F(3)$ and explain geometrically what you found. 22, net signed area $[1,3]$
   b. Find $F'(2)$. What does the sign of $F'(2)$ tell about $F(x)$? 9, pos., increasing @ $x=2$
   c. Find $F''(-2)$. What does the sign of $F''(-2)$ tell about $F(x)$? 12, concave up @ $x=-2$

6.6 2nd Fundamental Theorem of Calculus $\frac{d}{dx}\int_c^{g(x)} f(t)\,dt = f(g(x)) \bullet g'(x)$

In Exercises 6-9, calculate the derivative.

6. $\frac{d}{dx}\int_0^{x^2} \frac{t\,dt}{t+1}$ $\frac{2x^3}{x^2+1}$

7. $\frac{d}{dx}\int_1^{1/x} \cos^3 t\,dt$ $\frac{-\cos^3(1/x)}{x^2}$

8. $\frac{d}{ds}\int_{-6}^{\cos s} u^4\,du$ $-\sin s \cos^4 s$

9. $\frac{d}{dx}\int_1^{x^3} \frac{\sin t}{t}\,dt$ $\frac{3\sin(x^3)}{x}$

10. Find $G(1)$, $G(0)$, and $G(\pi/4)$, where $G(x) = \int_1^x \tan t\,dt$. 0, 0, 1

11. Find $H(-2)$ and $H(-2)$, where $H(x) = \int_{-2}^{x} \frac{du}{u^2+1}$. 0, 1/5

12. Let $F(x) = \int_2^x \frac{t+3}{t-1}\,dt$. Find $F(2)$, $F'(2)$, and $F''(2)$.
    0, 5, −4

13. Let $G(x) = \int_1^x (t^2 - 2)\,dt$. Calculate $G(1)$, $G'(1)$ and $G'(2)$. Then find the equation for $G(x)$. (solve the initial value problem) 0, −1, 2 $G(x) = \frac{x^3}{3} - 2x + \frac{5}{3}$

14. $F(x) = \int_2^x 4t^2 + 4\,dt$.
    a. Evaluate $F(2)$, $F'(2)$, and $F''(2)$.

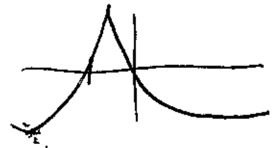
    0, 20, 16

6.6 Applications of 2nd Fundamental Theorem of Calculus $\frac{d}{dx}\int_{c}^{f(x)} f(t)\,dt$

15. $F(x)=\int_{2}^{x}\frac{t-2}{t+1}\,dt$ **Show all work.**

INC: $(-\infty,-1)\cup(2,+\infty)$
DEC: $(-1,2)$

a. Find the intervals where $F(x)$ is increasing and decreasing.
b. Find all critical points and stationary points for $F(x)$. $x=-1$ + 2
c. Find the concavity of $F(x)$. UP $(-\infty,+\infty)$
d. Find all the inflection points of $F(x)$. NONE
e. Include a sketch of $F(x)$ for *extra credit.*



16. For the given function, $F(x)=\int_{2}^{x} t^2+t-6\,dt$

a. Find $F(0)$ $22/3$
b. Find $F'(2)$ 0
c. Find the intervals of concavity. Down: $(-\infty,-1/2)$ Up: $(-1/2,+\infty)$
d. Identify the location of any inflection points. $x=-1/2$

17. $F(x)=\int_{2}^{x} 4t^2+4\ dt.$

a. Evaluate $F(2)$, $F'(2)$, *and* $F''(2)$. 0, 20, 16
b. Find all intervals of increase and decrease always increasing
c. Find all intervals of concavity Down $(-\infty,0)$; UP $(0,+\infty)$
d. Identify any stationary points, relative max/mins, and inflection points.
None None $x=0$

18. Let $A(x)=\int_{0}^{x} f(t)\,dt$ for $f(x)$ in **Figure 8.**

(a)

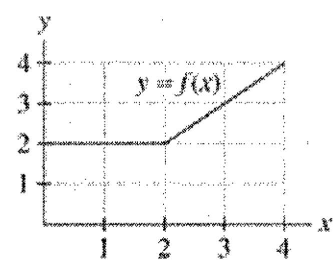
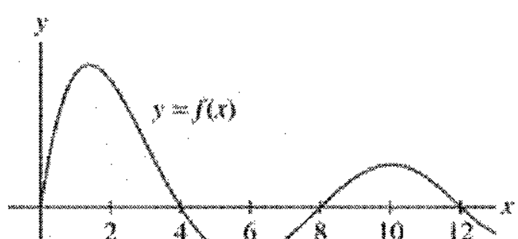


FIGURE 8
Calculate $A(2)$, $A(3)$, $A'(2)$, and $A'(3)$.
4, 6.5, 2, 3

19. Let $A(x)=\int_{0}^{x} f(t)\,dt$, with $f(x)$ given. Determine



(a) The intervals on which $A(x)$ is increasing and decreasing
INC: $(0,4)\cup(8,12)$
DEC: $(4,8)\cup(12,+\infty)$

(b) The values x where $A(x)$ has a local min or max
$x=8$ $x=4$ & 12

(c) The inflection points of $A(x)$ $x=2, 6, 10$

(d) The intervals where $A(x)$ is concave up or concave down
UP: $(0,2)\cup(6,10)$
Down: $(2,6)\cup(10,+\infty)$