If you're adding and subtracting diminishing numbers it's actually MUCH more likely that the sequence of partial sums (and thus the series) will converge.

Maybe think of a pendulum swing back and forth a little less each time. It will eventually be close enough to motionless to know where the stopping point will be.

However, the **Alternating Harmonic Series** actually converges by AST. It converges to $\ln 2$. You can see the proof on Youtube. Click here

See picture below

$a_n = (-1)^{n-1} b_n$ or

where b_n is a positive number. (In fact, $b_n = |a_n|$.)

The following test says that if the terms of an alternating series decrease toward 0 in absolute value, then the series converges.

The Alternating Series Test If the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + \cdots \qquad (b_n > 0)$$

satisfies

$$\text{(i)} \quad b_{n+1} \leq b_n \quad \text{for all } n$$

$$\text{(ii)} \quad \lim_{n \to \infty} b_n = 0$$

then the series is convergent.

Before giving the proof let's look at Figure 1, which gives a picture of the idea behind the proof. We first plot $s_1 = b_1$ on a number line. To find s_2 we subtract b_2, so s_2 is to the left of s_1. Then to find s_3 we add b_3, so s_3 is to the right of s_2. But, since $b_3 < b_2$, s_3 is to the left of s_1. Continuing in this manner, we see that the partial sums oscillate back and forth. Since $b_n \to 0$, the successive steps are becoming smaller and smaller. The even partial sums $s_2, s_4, s_6, \ldots$ are increasing and the odd partial sums $s_1, s_3, s_5, \ldots$ are decreasing. Thus, it seems plausible that both are converging to some number s, which is the sum of the series. Therefore, in the following proof we consider the even and odd partial sums separately.

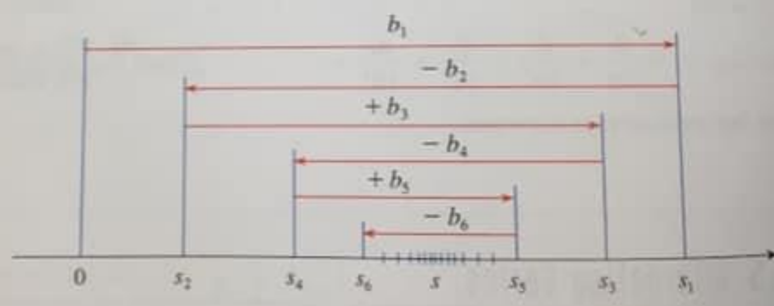

FIGURE 1

Proof of the Alternating Series Test We first consider the even partial sums:

$$s_2 = b_1 - b_2 \geq 0 \qquad \text{since } b_2 \leq b_1$$

$$s_4 = s_2 + (b_3 - b_4) \geq s_2 \qquad \text{since } b_4 \leq b_3$$

In general $\quad s_{2n} = s_{2n-2} + (b_{2n-1} - b_{2n}) \geq s_{2n-2} \qquad \text{since } b_{2n} \leq b_{2n-1}$

Thus $\quad 0 \leq s_2 \leq s_4 \leq s_6 \leq \cdots \leq s_{2n} \leq \cdots$

But we can also write

$$s_{2n} = b_1 - (b_2 - b_3) - (b \ldots) - b_{2n}$$