

Name Key

AP Calculus BC Integration By Parts

Find each of the following. Not all problems will require integration by parts. Be sure to show all work.

1.) $\int x \sin x \, dx$

$u = x \quad dv = \sin x$

$= -x \cos x + \int \cos x \, dx$

$= -x \cos x + \sin x + C$

2.) $\int x \cos(5x) \, dx$

$u = x \quad dv = \cos(5x)$

$= \frac{1}{5} x \sin(5x) - \frac{1}{5} \int \sin(5x) \, dx$

$= \frac{1}{5} x \sin(5x) + \frac{1}{25} \cos(5x) + C$

3.) $\int x e^{8x} \, dx$

$u = x \quad dv = e^{8x}$

$= \frac{1}{8} x e^{8x} - \frac{1}{8} \int e^{8x} \, dx$

$= \frac{1}{8} x e^{8x} - \frac{1}{64} e^{8x} + C$

4.) $\int \frac{(\ln x)^4}{x} \, dx$ (u-sub)

$= \frac{1}{5} (\ln x)^5 + C$

5.) ~~$\int x e^{x^2} \, dx$ (u-sub)~~

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~~$= \frac{1}{2} e^{x^2} + C$~~

6.) $\int \sqrt{x} \ln x \, dx$

$u = \ln x \quad dv = \sqrt{x}$

$= \frac{2}{3} x^{3/2} \ln x - \int \frac{2/3 x^{3/2}}{x} \, dx$

$= \frac{2}{3} x^{3/2} \ln x - \frac{2}{3} \int x^{1/2} \, dx$

$= \frac{2}{3} x^{3/2} \ln x - \frac{4}{9} x^{3/2} + C$

$$7.) \int (6x+2)e^{-3x} dx$$

$$u = 6x+2 \quad dv = e^{-3x}$$

$$= -\frac{1}{3}(6x+2)e^{-3x} - \frac{2}{3}e^{-3x} + C$$

$$8.) \int x^3 \ln x dx$$

$$u = \ln x \quad dv = x^3$$

$$= \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$$

$$9.) \int x\sqrt{x+1} dx \quad (u\text{-sub})$$

$$u = x+1 \rightarrow x = u-1$$

$$= \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$$

$$10.) \int \frac{4x}{e^{2x}} dx$$

$$u = 4x \quad dv = e^{-2x}$$

$$= -\frac{1}{2} \cdot 4xe^{-2x} + \frac{4}{2} \int e^{-2x} dx$$

$$= -2xe^{-2x} - e^{-2x} + C$$

$$11.) \int_0^1 x \ln(1+x^2) dx$$

$$u = \ln(1+x^2) \quad dv = x$$

$$= \frac{1}{2}x^2 \ln(1+x^2) - \int \frac{x^3}{1+x^2} dx$$

(long division)

$$= \ln(2) - \frac{1}{2}$$

$$12.) \int_1^e 4x \ln x dx$$

$$u = \ln x \quad dv = 4x$$

$$= 2x^2 \ln x - \int \frac{2x^2}{x} dx$$

$$= 2x^2 \ln x - \int 2x dx$$

$$= 2x^2 \ln x - x^2 \Big|_1^e$$

$$= (2e^2 \ln e - e^2) - (2 \ln 1 - 1)$$

$$= e^2 + 1$$