Name ________________________________

AP Calculus BC

7.5 Eulers Method Use calculator as needed

1. Given $\frac{dy}{dx} = 2x + 3$ and $(1,3)$, approximate $f(1.4)$ using $\Delta x = 0.2$

x	y	$\Delta y = (2x+3)\cdot(0.2)$	New y
1	3	$\Delta y = 5 \cdot 0.2 = 1$	$3 + 1 = 4$
1.2	4	$\Delta y = [2(1.2)+3]\cdot(0.2) = 1.08$	$4 + 1.08 = 5.08$
1.4	5.08		

2. Given $f'(x,y) = \frac{x}{y}$ and $(0,3)$, approximate $f(0.3)$ using $\Delta x = 0.1$

x	y	$\Delta y = \frac{x}{y}\cdot(0.1)$	New y
0	3	$= \left(\frac{0}{3}\right)\cdot .1 = 0$	$3 + 0 = 3$
.1	3	$= \left(\frac{.1}{3}\right)(.1) = .00\overline{3}$	$3 + .00\overline{3} = 3.00\overline{3}$
.2	$3.00\overline{3}$	$= \left(\frac{.2}{3.00\overline{3}}\right)(.1) = .0665$	$3.00\overline{3} + .0665 = 3.0699$
.3	~~3.0699~~		

3.00699

3. Given $f'(x,y) = y - 1$ and $f(0) = 2$ approximate $f(1.2)$, using 4 equal steps.

$\Delta x = \frac{1.2 - 0}{4} = .3$

x	y	$\Delta y = (y-1)\cdot(.3)$	New y
0	2	$= (2-1)(.3) = .3$	$2 + .3 = 2.3$
.3	2.3	$= (2.3-1)(.3) = .39$	$2.3 + .39 = 2.69$
.6	2.69	$= (2.69-1)(.3) = .507$	$2.69 + .507 = 3.197$
.9	3.197	$= (3.197-1)(.3) = .6591$	$3.197 + .6591 = 3.8561$
1.2	3.8561		

4. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 2x + y$ with initial condition $f(1) = 0$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length, starting at $x = 1$?

A. 0

B. 1

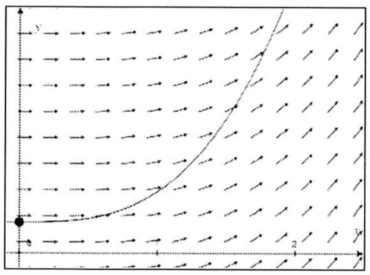
C. 2.75

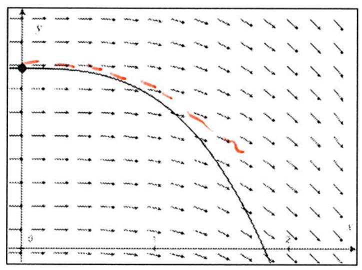
D. 3

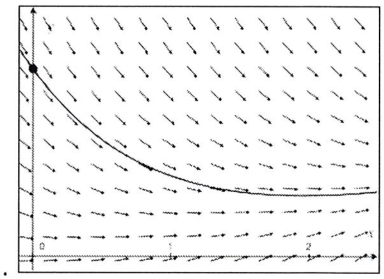
$\Delta x = \frac{2-1}{2} = .5$

x	y	$\Delta y = (2x+y)(.5)$	New y
1	0	$= (2\cdot 1 + 0)(.5) = 1$	$0 + 1 = 1$
1.5	1	$= [2(1.5)+1](.5) = 2$	$1 + 2 = 3$
2	(3)		

5. The slope field for a differential equation and the particular solution passing through the point on the y-axis are shown. If Euler's method is used to approximate the solution, which graph will give the over-approximation?

I. 

II. 

III. 

A. I only B. II only C. III only D. I and III only

6. Answer the following questions.

a. Given the differential equation $\frac{dy}{dx} = x + 2$ and $y(0) = 3$. Find an approximation for $y(1)$ by using Euler's Method with two equal steps.

$\Delta x = \frac{1-0}{2} = .5$

x	y	$\Delta y = (x+2)(.5)$	New y
0	3	$= (0+2)(.5) = 1$	$3 + 1 = 4$
.5	4	$= (.5+2)(.5) = 1.25$	$4 + 1.25 = 5.25$
1	(5.25)		

b. (Review of FTC) Solve the differential equation $\frac{dy}{dx} = x + 2$ with the initial condition $y(0) = 3$, and use your solution to find $y(1)$.

$y = 3 + \int_0^x (t+2)\,dt$

$y = 3 + \frac{t^2}{2} + 2t \Big|_0^x$

$y = 3 + \left[\left(\frac{x^2}{2} + 2x\right) - 0\right]$

$y = \frac{x^2}{2} + 2x + 3$

$y(1) = \frac{1^2}{2} + 2(1) + 3 = 5.5$

c. The error in using Euler's Method is the difference between the approximate value and the exact value. What was the error in your answer? How could you produce a smaller error using Euler's Method?

#6a Estmate 5.25

#6b Actual 5.5

Error = 5.5 − 5.25 = .25

Use smaller steps in 6a. for more accuracy