

Fruit Fly Population

1. An absent-minded graduate student at a university is studying a population of fruit flies in a biology lab. She works under the premise that this experimental population of fruit flies increases according to the law of exponential growth. She counts 100 flies after the second day of the experiment and 300 flies after the fourth day, however, she forgot to record the number of fruit-flies she initially had at the beginning of the experiment. Approximately how many flies were in the original population? Round to the nearest fly.

$$P(t) = Ce^{kt}, \quad (2, 100) + (4, 300)$$

$$100 = Ce^{k \cdot 2}$$

$$C = \frac{100}{e^{2k}} \quad (*k \text{ was not eliminated because } t \neq 0) \therefore \text{use 2nd pt}$$

$$300 = Ce^{k \cdot 4}$$

$$C = \frac{300}{e^{4k}}$$

$$\frac{300}{e^{4k}} = \frac{100}{e^{2k}}$$

$$\rightarrow 300e^{2k} = 100e^{4k}$$

$$\frac{300}{100} = \frac{e^{4k}}{e^{2k}}$$

$$3 = e^{2k}$$

$$\ln 3 = 2k$$

$$k = \frac{1}{2} \ln 3$$

$$C = \frac{100}{e^{2 \cdot (\frac{1}{2} \ln 3)}} = \frac{100}{3}$$

$$P(t) = \frac{100}{3} e^{\frac{t}{2} \ln 3}$$

$$= \frac{100}{3} e^{\ln 3^{t/2}}$$

$$= \frac{100}{3} \cdot 3^{t/2}$$

$$P(0) = \frac{100}{3} \cdot 3^0 = 100/3 \text{ fruit flies}$$

Advertising Problem

2. Four months after it stops advertising, a manufacturing company notices that its sales have dropped from 100,000 units per month to 80,000 units per month. If the sales follow an exponential pattern of decline, what will they be, to the nearest unit, after another 2 months?

$$P(t) = Ce^{kt}; \quad (0, 100,000) + (4, 80,000)$$

$$100,000 = Ce^{k \cdot 0}$$

$$C = 100,000$$

$$P(t) = 100,000 e^{kt}$$

$$80,000 = 100,000 e^{k(4)}$$

$$\frac{4}{5} = e^{4k}$$

$$\ln\left(\frac{4}{5}\right) = 4k$$

$$k = \frac{1}{4} \ln\left(\frac{4}{5}\right)$$

$$P(t) = 100,000 e^{\frac{t}{4} \ln\left(\frac{4}{5}\right)}$$

$$= 100,000 e^{\ln\left(\frac{4}{5}\right)^{t/4}}$$

$$= 100,000 \cdot \left(\frac{4}{5}\right)^{t/4}$$

$$P(6) = 100,000 \cdot \left(\frac{4}{5}\right)^{6/4}$$

$$\approx 71554.175 \text{ units}$$