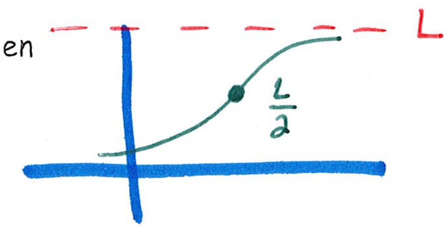
If y is a differentiable function of t such that $\frac{dP}{dt} = kP(L-P)$, then



- $P = \dfrac{L}{1+Ce^{-Lkt}}$
- $\lim_{t\to\infty} P(t) = L$ (L is the carrying capacity.)
- $\frac{dP}{dt}$ is at its maximum (the rate of growth is increasing the fastest) when the function reaches half its carrying capacity, $\frac{L}{2}$. (this is also the point of inflection on $P(t)$)

---

1. The population, P of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where $P(0) = 3000$ and t is the time in years. What is $\lim_{t\to\infty} P(t)$?

$$\frac{dP}{dt} = \frac{P}{5000}(10{,}000 - P)$$

$$\lim_{t\to\infty} P(t) = L$$
$$= 10{,}000$$

2. A rumor spreads at the rate $\frac{dP}{dt} = 2P(1-P)$ where P is the portion of the population that has heard the rumor at time t. What portion of the population has heard the rumor when it is spreading the fastest?

$L = 1$

$\frac{dP}{dt}$ is at a max when $P = \frac{L}{2}$ or $\frac{1}{2}$

1/2 the population heard the rumor when it was spreading the fastest