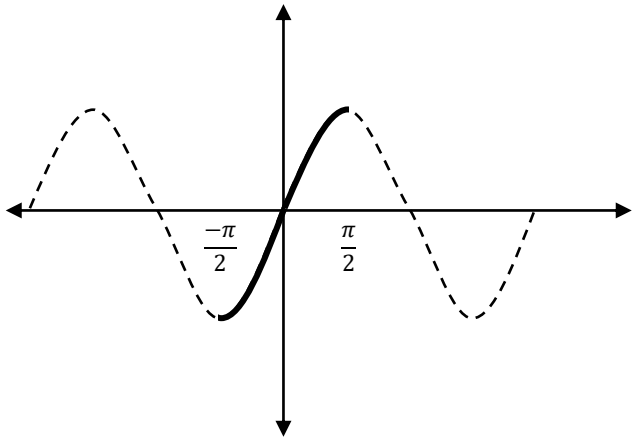
Inverse Trigonometric Functions

Inverse Sine Function

To have an inverse a function must be a **one-to-one** function.

If we restrict the domain of the function $y = \sin x$ to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ the function is now a one-to-one function.



The inverse sine function is:

$y = \arcsin x$ iff $\sin y = x$ (or $y = \sin^{-1} x$)

where $-1 \leq x \leq 1$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

Examples (using your memorized unit circle)

Find: $\arcsin(-1)$

Find: $\arcsin\left(\frac{1}{2}\right)$

Find: $\arcsin\left(\frac{\sqrt{3}}{2}\right)$

Definitions of the Inverse Trigonometric Functions

Function	Domain	Range
$y = \arcsin x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \arccos x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \arctan x$	$-\infty \leq x \leq \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Find the exact values (using your memorized unit circle)

$\arccos \frac{\sqrt{3}}{2} =$

$\cos^{-1}(-0.5) =$

$\arctan\ 1 =$

$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) =$

$\sin^{-1}\left(-\frac{1}{2}\right) =$

$\tan^{-1}(-1) =$

Calculators and Inverse Trigonometric Functions (mode = Radian)

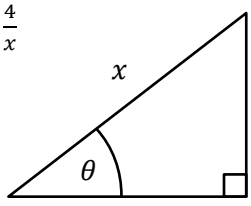
$\arctan\ 4.84 \approx$

$\cos^{-1}(-0.349) \approx$

$\arcsin(-1.1) \approx$

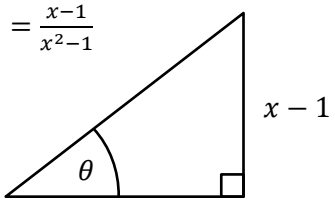
Drawing sketches to represent angles

$\cos\theta = \frac{4}{x}$



$\theta = \arccos\left(\frac{4}{x}\right)$

$\tan\theta = \frac{x-1}{x^2-1}$



$\theta = \arctan\left(\frac{x-1}{x^2-1}\right)$

Composites of Functions

If $-1 \leq x \leq 1$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ then $\sin(\arcsin x) = x$ and $\arcsin(\sin y) = y$

If $-1 \leq x \leq 1$ and $0 \leq y \leq \pi$ then $\cos(\arccos x) = x$ and $\arccos(\cos y) = y$

If x is a real number and $-\frac{\pi}{2} < y < \frac{\pi}{2}$ then $\tan(\arctan x) = x$ and $\arctan(\tan y) = y$

Examples:

$\tan(\arctan(-14)) = -14$

$\sin(\arcsin \pi)$ = no solution because π is not between $-1 \leq x \leq 1$

$\cos(\arccos 0.54) = 0.54$

Evaluating Composites of Trigonometric Functions

Examples:

$\cos\left(\arctan\left(-\frac{3}{4}\right)\right)$

$\cos\left(\sin^{-1}\frac{2}{3}\right)$

Write each of the following as an algebraic expression in x.

$\tan(\arccos 2x)$

$\sec(\tan^{-1} x)$

Evaluate the following.

a) $\cos(\arctan\sqrt{3} + \arcsin\frac{1}{3})$

b) $\tan(2\arcsin\frac{2}{5})$

Write $\sin(tan^{-1}\omega)$ as an expression in ω.