

This will give you an idea of the types of problems you can expect to see on the midterm and the general difficulty of the material. While not every concept in the actual exam is represented here, most of them are.

**Limits**

1.  $\lim_{x \rightarrow 2} \frac{-3x-6}{x^2+x-2}$  is

A. -1

B.  $-\frac{1}{3}$

C.  $\frac{1}{3}$

D. 1

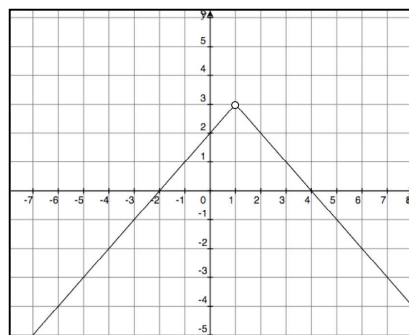
2. The figure to the right shows the graph of  $f(x)$ .

Which of the following statements are true?

I.  $\lim_{x \rightarrow 1^-} f(x)$  exists

II.  $\lim_{x \rightarrow 1^+} f(x)$  exists

III.  $\lim_{x \rightarrow 1} f(x)$  exists



A. I only

B. II only

C. I and II only

D. I, II and III

3. Let  $f(x) = \begin{cases} \frac{x^2 - 6x + 9}{x - 3} & \text{if } x \neq 3 \\ \sqrt{3x} & \text{if } x = 3 \end{cases}$

Which of the following statements I, II, and III are true?

I.  $\lim_{x \rightarrow 3} f(x)$  exists

II.  $f(3)$  exists

III.  $f$  is continuous at  $x = 3$

A. only I

B. only II

C. I and II

D. none of them

4.  $\lim_{x \rightarrow -\infty} \frac{-4 - x - x^2}{2x^2 + 3x - 2} =$  is

A. 0

B.  $\infty$

C. -2

D.  $-\frac{1}{2}$

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### Derivatives

5. If  $f(x) = (1 - 4x^2)^4$ , then  $f'(x) =$

A.  $-32x(1 - 4x^2)^3$

B.  $4(1 - 8x)^3$

C.  $-32(1 - 8x)^3$

D.  $4(1 - 4x^2)^3$

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6. The slope of the tangent line to  $y = \frac{x}{2x-1}$  at  $x = 4$  is

A.  $\frac{1}{2}$

B.  $-\frac{1}{14}$

C.  $\frac{1}{14}$

D.  $-\frac{1}{49}$

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7. The second derivative of  $f(x) = \sin x \cos x$  is

A. 0

B.  $-4 \sin x \cos x$

C.  $(\cos x)^2 - (\sin x)^2$

D.  $2 \cos x - 2 \sin x$

8. If  $f(-2) = 4$  and  $f'(-2) = -1$ , find the derivative of  $\frac{f(x)}{x^2}$  at  $x = -2$ .

A.  $\frac{3}{4}$

B. 1

C.  $\frac{1}{4}$

D.  $-\frac{1}{4}$

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9. Find the equation of the line tangent to  $y = \sec x$  at  $x = \frac{\pi}{4}$ .

A.  $y - \frac{\sqrt{2}}{2} = \sqrt{2}\left(x - \frac{\pi}{4}\right)$

B.  $y - \sqrt{2} = \sqrt{2}\left(x - \frac{\pi}{4}\right)$

C.  $y = \sqrt{2}\left(x - \frac{\pi}{4}\right)$

D.  $y - \sqrt{2} = x - \frac{\pi}{4}$

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10. For how many values of  $x$  are the tangent lines to  $y = \sin x + \cos x$  and  $y = \frac{x^3}{12} - \frac{x^2}{2} - \frac{x}{4}$  parallel?

A. 1

B. 2

C. 3

D. 4

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11. Find the linear approximation to  $f\left(\frac{1}{2}\right)$  for  $f(x) = e^{\sin x + \cos x}$  at  $x = 0$ .

12. Let  $f$ ,  $g$  and their derivatives be defined by the table below. If the derivatives of  $f(g(x))$  and  $g(f(x))$  are equal at  $x = 3$ , what is the value of  $a$ ?

$x$	1	2	3	4
$f(x)$	3	2	1	4
$g(x)$	2	1	4	3
$f'(x)$	4	3	4	2
$g'(x)$	$a$	1	2	3

- A. 1                                      B. 2                                      C. 3                                      D. 4

13. A function  $f$  and its derivative take on the values shown in the table. If  $g$  is the inverse of  $f$ , find  $g'(b)$ .

- a.  $-a$                                       b.  $-c$                                       c.  $\frac{1}{a}$                                       d.  $\frac{1}{c}$

### Implicit Differentiation

14. If  $x^2 + xy - y = 7$ , find  $\frac{dy}{dx}$  at  $(3, -1)$

- a. 1                                      b.  $-\frac{5}{2}$                                       c.  $-1$                                       d.  $\frac{1}{2}$

15. At which points is the tangent line to the curve  $8x^2 + 2y^2 = 6xy + 14$  vertical?

- I.  $(-2, -3)$                                       II.  $(3, 8)$                                       III.  $(4, 6)$
- a. I only                                      b. II only                                      c. III only                                      d. I and III only

16. For  $\ln xy + 3x = 4$ , find the equation of the line tangent to the curve at  $(1, e)$ .

A.  $y = -3ex + 4e$

B.  $y = -3ex + 2e$

C.  $y = -4ex + 5e$

D.  $y = -4ex + 3e$

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### Continuity and Differentiability

17. If  $f(x) = \begin{cases} x^2 - 5x + 8, & x \geq 3 \\ x - 1, & x < 3 \end{cases}$ , describe the behavior of the graph of  $f(x)$  at  $x = 3$

I.  $\lim_{x \rightarrow 3} f(x)$  exists

II.  $f(x)$  is continuous at  $x = 3$

III.  $f(x)$  is differentiable at  $x = 3$

a. I only

b. II only

c. I and II only

d. I, II and III

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### Related Rates

18. Matthew is visiting Gregory at his home on North Street. Shortly after Matthew leaves, Gregory realizes that Matthew left his wallet and begins to chase him. When Gregory is 3 miles from the  $90^\circ$  intersection along North Street traveling at 45 mph towards the intersection, Matthew is 4 miles along East street traveling away from the intersection at 30 mph. At that time, how fast is the distance between the two men changing?



A. getting closer at 3 mph

B. getting further away at 51 mph

C. getting closer at 51 mph

D. Getting closer at 15 mph

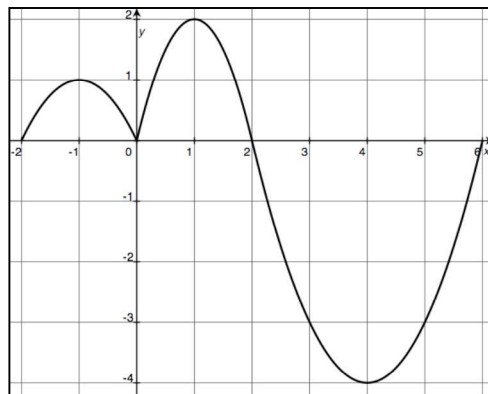
19. A cylinder has both its height and radius changing. Its height is increasing at the rate of 3 meters/min. When the height of the cylinder is 8 meters and its radius is 2 meters, the volume is not changing. How fast is the radius decreasing in meters/min? (The volume of a cylinder is given by  $V = \pi r^2 h$ ).
- A. 3                                      B.  $\frac{3}{8}$                                       C. 6                                      D.  $3\pi$
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### Function Analysis

20. Given that  $f(x) = x^3$  find all values of  $x$  in the interval  $[-1, 1]$  that satisfy the mean value theorem.
- A. 0                                      B.  $\sqrt{\frac{1}{3}}$                                       C.  $\pm\sqrt{\frac{1}{3}}$                                       D.  $\pm\frac{1}{9}$
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21. The graph of  $f'$ , the derivative of  $f$  is shown to the right for  $-2 \leq x \leq 6$ . At what values of  $x$  does  $f$  have a horizontal tangent line?

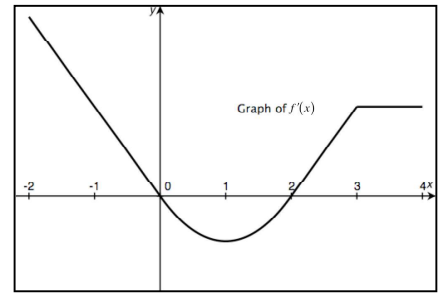
- A.  $x = 0$  and 2 only  
B.  $x = -1, x = 1, x = 4$   
C.  $x = -2, x = 2, x = 6$   
D.  $x = -2, x = 0, x = 2, x = 6$



22. Given  $f(x) = 2 - \frac{x^3}{6} - x^2$ . On what interval(s) is the graph of  $f$  concave upwards?

- A.  $(-\infty, -2)$                                       B.  $(-\infty, 2)$                                       C.  $(-2, \infty)$                                       D.  $(2, \infty)$
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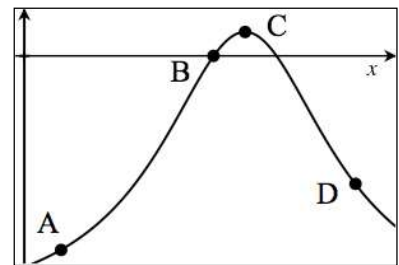
23. The graph of  $f'(x)$ , the derivative of  $f$ , is shown to the right. Which of the following statements is **not** true?



- A.  $f$  is increasing on  $2 < x \leq 4$ .
- B.  $f$  has a local minimum at  $x = 1$ .
- C.  $f$  has a local maximum at  $x = 0$ .
- D.  $f$  has an inflection point at  $x = 1$ .

24. The graph of  $y = f(x)$  is to the right. At what point are both

$$\frac{dy}{dx} > 0 \text{ and } \frac{d^2y}{dx^2} > 0 ?$$



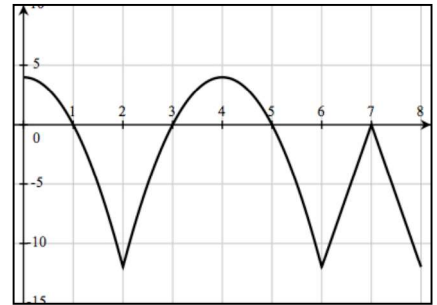
25. If  $f(x) = \frac{1}{x^n}$ , where  $n > 0$  and  $x > 0$ , describe the concavity of the graph of  $f(x)$ .

- A. Always concave up
- B. Always concave down
- C. Concave down if  $n < 1$ , concave up if  $n \geq 1$
- D. Concave up if  $n < 1$ , concave down if  $n \geq 1$

## Straight-Line Motion

26. A locomotive is moving along a straight track. Its velocity  $v$  of the locomotive at time  $t$ ,  $0 \leq t \leq 8$  is given by the function whose graph is to the right. At what value of  $t$  does the locomotive change direction?

- A. 4 only  
B. 2, 4 and 6 only  
C. 1, 3, and 5 only  
D. 1, 3, 5 and 7 only



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27. A particle moves along a horizontal line with position  $x(t) = \frac{10}{t}$ . Describe its motion at  $t = 1$ .

- A. Moving right and slowing down  
B. Moving right and speeding up  
C. Moving left and slowing down  
D. Moving left and speeding up

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28. A ball is thrown straight up from the top of a hill 30 feet high with initial velocity of 72 ft/sec. How high above level ground will the ball get? (objects subjected to gravity adhere to  $s(t) = -16t^2 + v_0t + s_0$  where  $s$  is the height of the object in feet,  $v_0$  is the initial velocity and  $s_0$  is the initial height).

- A. 72 feet  
B. 88 feet  
C. 111 feet  
D. 144 ft



## Absolute Extrema

29. If  $f(x) = \frac{x^3}{3} - x^2$  on  $[-2, 3]$ ,  $f(x)$  has an absolute minimum at

A.  $x = -2$

B.  $x = 0$

C.  $x = 2$

D.  $x = 0$  and  $x = 2$

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30. If the sum of two numbers  $x$  and  $y$  is 12, what is the maximum product of  $x^3y$ ?

A. 27

B. 729

C. 2,187

D. 19,683

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31.  $f(x) = \frac{x^2}{4} + x - 4\cos(2x)$  on  $[0, 2\pi]$  has a possible maximum slope at the  $x$ -value that solves the equation

A.  $\frac{x}{2} + 1 - 8\sin(2x) = 0$

B.  $\frac{x}{2} + 1 + 8\sin(2x) = 0$

C.  $\frac{1}{2} + 16\cos(2x) = 0$

D.  $\frac{1}{2} + 4\cos(2x) = 0$

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## L'Hospital's Rule

32. Find  $\lim_{x \rightarrow 1} \frac{\sqrt{x} \ln x}{x^2 - 1}$

A. 0

B.  $\frac{1}{2}$

C. 1

D. undefined