



Trigonometric Equations

A conditional statement is an equation in which some replacements for the variable make the statement true and some replacements for the variable make it false. For example:

$$2x+3=5$$

$$x^2-5x=10$$

$$2x=8$$

Today, we will be discussing conditional equations that involve trigonometric functions.

Examples: Solve the following

**Never divide by a common factor*
to eliminate \rightarrow only rearrange*

1. $3\sin x = \sqrt{3} + \sin x$

$$2\sin x = \sqrt{3}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$x = 60^\circ + 360^\circ n$$

$$120^\circ + 360^\circ n$$

or $\frac{\pi}{3} + 2\pi n$

$$\frac{2\pi}{3} + 2\pi n$$

2. $\sin x + \cos x = 0$ for $0^\circ \leq x < 360^\circ$

$$\sin x = -\cos x$$

$$\sin^2 x = \cos^2 x$$

$$1 - \cos^2 x = \cos^2 x$$

$$1 = 2\cos^2 x$$

$$\cos^2 x = \frac{1}{2}$$

$$\cos x = \pm \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$$

$$x = \cancel{45^\circ}, \cancel{135^\circ}, \cancel{225^\circ}, \cancel{315^\circ}$$

extraneous solutions

**This will only happen in Q_{II} & Q_{IV}*

3. $\sin x \tan x = \sin x$ for $0^\circ \leq x < 360^\circ$

$$\sin x \tan x - \sin x = 0$$

$$\sin x (\tan x - 1) = 0$$

$$\sin x = 0 \quad \tan x = 1$$

$$x = 0^\circ, 180^\circ, 45^\circ, 225^\circ$$

4. $\tan x + \sqrt{3} = \sec x$ for $0 \leq x < 2\pi$

$$(\tan x + \sqrt{3})^2 = \sec^2 x$$

$$\tan^2 x + 2\sqrt{3}\tan x + 3 = 1 + \tan^2 x$$

$$2\sqrt{3}\tan x = -2$$

$$\tan x = \frac{-2}{2\sqrt{3}} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

$$x = \cancel{\frac{5\pi}{6}}, \frac{11\pi}{6}$$

5. $\tan^2 x + \tan x - 2 = 0$ for $0 \leq x < 2\pi$

$$(\tan x + 2)(\tan x - 1) = 0$$

$$\tan x = -2 \quad \tan x = 1$$

$$\theta' = 1.107$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$



$$\pi - \theta' = 2.034$$

$$2\pi - \theta' = 5.176$$

6. $\cot^2 x + 3\cot x - 1 = 1$ for $0^\circ \leq x < 360^\circ$

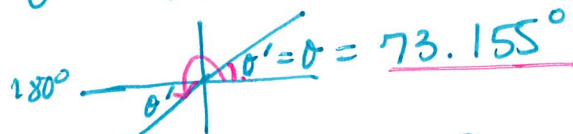
$$\cot^2 x + 3\cot x - 1 = 0$$

$$\cot x = \frac{-3 \pm \sqrt{9 - 4(1)(-1)}}{2(1)}$$

$$\cot x = \frac{-3 + \sqrt{13}}{2}$$

$$\tan x = \frac{2}{-3 + \sqrt{13}} > 0$$

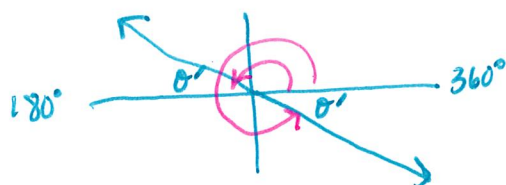
$$\theta' = 73.155^\circ$$



$$180^\circ + \theta' = 253.155^\circ$$

$$\tan x = \frac{-2}{3 - \sqrt{13}} < 0$$

$$\theta' = 16.845^\circ$$



$$180^\circ - \theta' = 163.155^\circ$$

$$360^\circ - \theta' = 343.155^\circ$$

$$x = 73.155^\circ, 163.155^\circ$$

$$253.155^\circ, 343.155^\circ$$