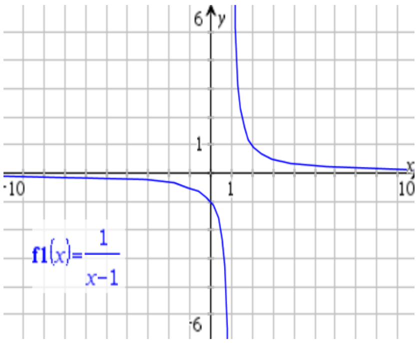
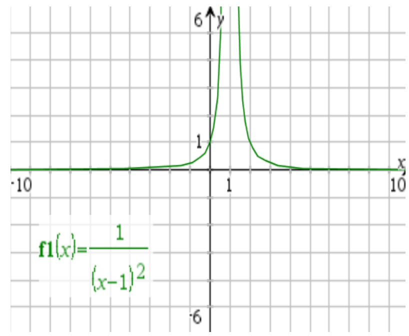
LIM	AP CALCULUS AB	
3	Topics: 1.14	Connecting Infinite Limits and Vertical Asymptotes
Learning Objective LIM-2.D: Interpret the behavior of functions using limits involving infinity.		

Infinite Limits

Consider the following functions:

$f(x) = \frac{1}{x-1}$ and $g(x) = \frac{1}{(x-1)^2}$





Example 1: Find each limit and confirm the result with the graphs above.

a. $\lim_{x \to 1^-} f(x) =$

d. $\lim_{x \to 1^-} g(x) =$

b. $\lim_{x \to 1^+} f(x) =$

e. $\lim_{x \to 1^+} g(x) =$

c. $\lim_{x \to 1} f(x) =$

f. $\lim_{x \to 1} g(x) =$

What do limits that produce results of infinity or negative infinity say about the function?

Vertical Asymptotes

Definition of Vertical Asymptotes

If $\lim_{x \to c^-} f(x) = \infty$ or $-\infty$ or $\lim_{x \to c^+} f(x) = \infty$ or $-\infty$, then $x = c$ is a vertical asymptote.

TIP. It is likely that by now, you have discovered that for fairly simple rational functions, finding the location of any vertical asymptote can be done without actually finding the value, c, that x should approach to ensure a limit that approaches either positive ∞ or negative ∞.
It is possible to find the asymptotes, by reducing the rational function to its simplest form and then setting any denominator factors equal to zero.

LIM	AP CALCULUS AB	
2	Topics: 1.15	Connecting Limits at Infinity and Horizontal Asymptotes
Learning Objective LIM-2.D: Interpret the behavior of functions using limits involving infinity.		

Consider the following function: $f(x) = \frac{3x^2}{x^2+1}$

For convenience, a table of values for the above function would yield the following:

x	$-\infty$	-100	-10	-1	0	1	10	100	∞
$f(x)$	3	2.99997	2.97	1.5	0	1.5	2.97	2.9997	3

Fill in the blanks below.
As $x \to \infty$ $f(x) \to$ _____ and
as $x \to -\infty$ $f(x) \to$ _____

Therefore, we can say what pair of limit statements?

____

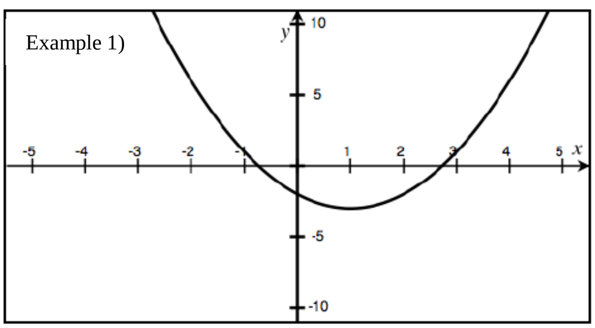
____

Limits At Infinity

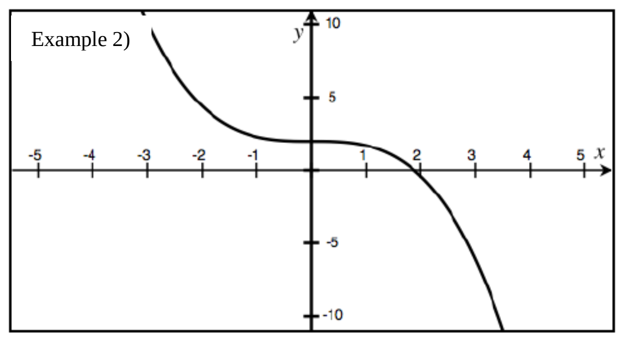
The concept of limits as x approaches infinity means the following: "what happens to y as x gets infinitely large." We are interested in what is happening to the y-value as the curve gets farther and farther to the right. We can also talk about limits as x approaches negative infinity. This means what is happening to the y-value as the curve gets farther and farther to the left.

The terminology we use are the following: $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
Note that it makes no sense to talk about $\lim_{x\to\infty^+} f(x)$ and $\lim_{x\to-\infty^-} f(x)$.

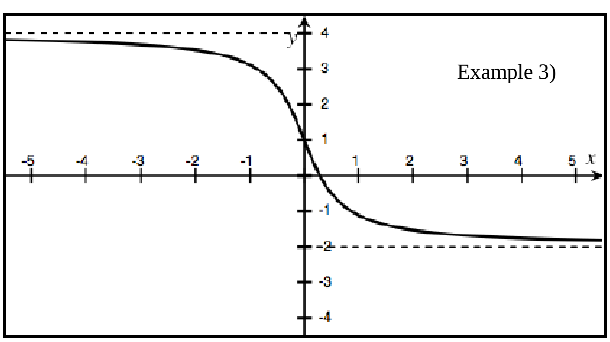
Examples 1-5: For each graph of $f(x)$, find the given limit.



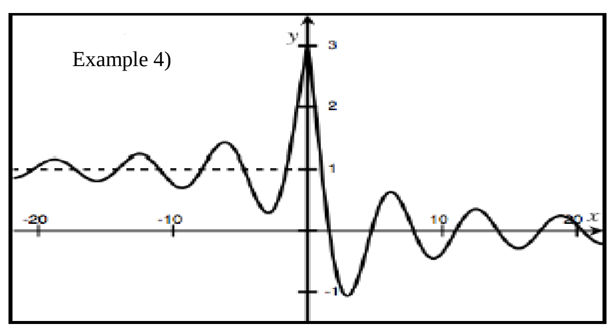
a.) $\lim_{x\to-\infty} f(x) =$ _____ **b.)** $\lim_{x\to\infty} f(x) =$ _____



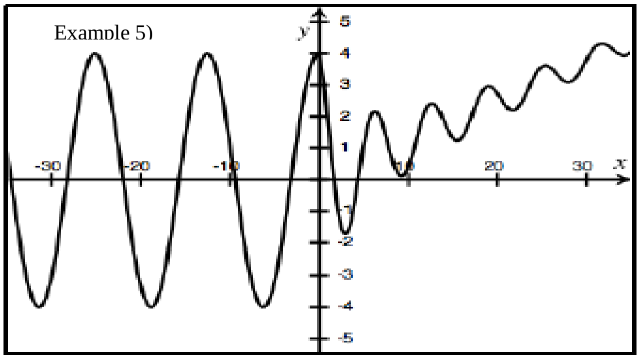
a.) $\lim_{x\to-\infty} f(x) =$ _____ **b.)** $\lim_{x\to\infty} f(x) =$ _____



a.) $\lim_{x\to-\infty} f(x) =$ _____ **b.)** $\lim_{x\to\infty} f(x) =$ _____



a.) $\lim_{x\to-\infty} f(x) =$ _____ **b.)** $\lim_{x\to\infty} f(x) =$ _____



A Video for some problems very similar to Examples 1-5 can be seen by scanning the QR Code.

a.) $\lim\limits_{x \to -\infty} f(x) =$ _____ **b.)** $\lim\limits_{x \to \infty} f(x) =$ _____

Example 6: **Evaluating a Limit at Infinity**

Evaluate $\lim\limits_{x \to \infty}\left(5 - \dfrac{2}{x^2}\right)$

> **Limit at Infinity Theorem**
>
> If $r > 0$ is a rational number, then
> $$\lim_{x \to \infty} \frac{1}{x^r} = 0.$$
> If $r > 0$ is a rational number, such that x^r is defined for all x, then
> $$\lim_{x \to -\infty} \frac{1}{x^r} = 0.$$

Example 7: **Evaluating a Limit at Infinity (Algebraically)**

Evaluate $\lim\limits_{x \to \infty} \dfrac{2x - 1}{x + 1}$

Note: Would it have made any difference if x approached $-\infty$?

Do you remember a shortcut from Math Analysis?

Guidelines for Finding Limits at $\pm\infty$ of Rational Functions

1. If the degree of the numerator is LESS THAN the degree of the denominator, then the limit of the rational function is __________________.
2. If the degree of the numerator is EQUAL TO the degree of the denominator, then the limit of the rational function is ____.
3. If the degree of the numerator is GREATER THAN the degree of the denominator, then the limit of the rational function ____.

Horizontal Asymptotes

Definition of a Horizontal Asymptote
The line $y = L$ is a **horizontal asymptote** of the graph of $f(x)$ if $\lim_{x\to-\infty} f(x) = L$ or $\lim_{x\to\infty} f(x) = L$

Example 8: **A Comparison of Three Rational Functions**
Find each of the following limits.

a. $\lim_{x\to\infty} \frac{2x+5}{3x^2+1}$

b. $\lim_{x\to\infty} \frac{2x^2+5}{3x^2+1}$

c. $\lim_{x\to\infty} \frac{2x^3+5}{3x^2+1}$

Example 9: **A Function Where the Results Differ**
Find each of the following limits.

a. $\lim_{x\to\infty} \frac{3x-2}{\sqrt{2x^2+1}}$

b. $\lim_{x\to-\infty} \frac{3x-2}{\sqrt{2x^2+1}}$

Trig Limits to Memorize

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{x\to 0} \frac{\sin bx}{bx} = 1$

$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$

Example 10:

$$\lim_{x\to 0} \frac{\sin 5x}{4x} =$$