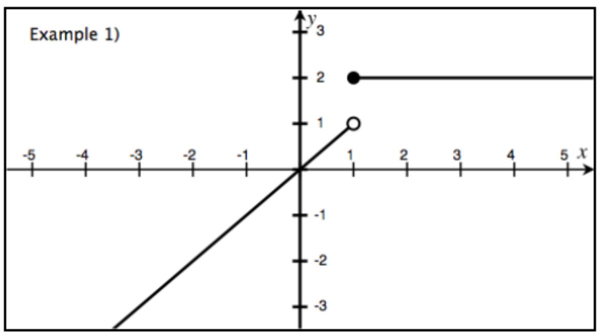
LIM	AP CALCULUS AB	
2	Topic: 1.3	Estimating Limit Values from Graphs
Learning Objective LIM-1.C: Estimate limits of functions.		

Limit Notation and One-Sided Limits

- If we want to find the limit of $f(x)$ as x approaches some value of c from the left hand side, we will write $\lim\limits_{x \to c^-} f(x)$.
- If we want to find the limit of $f(x)$ as x approaches some value of c from the right hand side, we will write $\lim\limits_{x \to c^+} f(x)$.
- In order for a limit to exist at c, $\lim\limits_{x \to c^-} f(x)$ must equal $\lim\limits_{x \to c^+} f(x)$ and we say $\lim\limits_{x \to c} f(x) = L$.

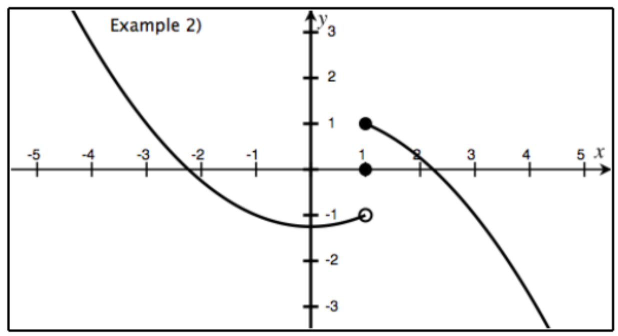
In order for a limit to exist, the function must be approaching the same y-value as the x approaches some value c from either the left or the right side. If this does not happen, we say that the limit does not exist (DNE) as x approaches c.

<u>Examples 1-7</u>: For each graph of $f(x)$, find the required information.



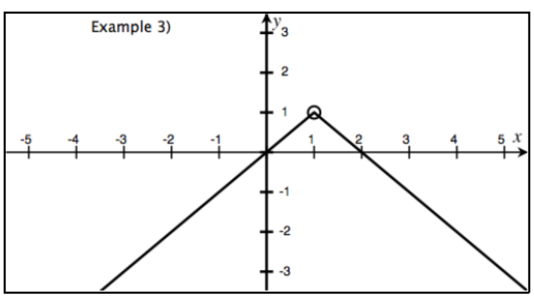
a.) $\lim\limits_{x \to 1^-} f(x) =$ _____ **b.)** $\lim\limits_{x \to 1^+} f(x) =$ _____

c.) $\lim\limits_{x \to 1} f(x) =$ _____ **d.)** $f(1) =$ _____

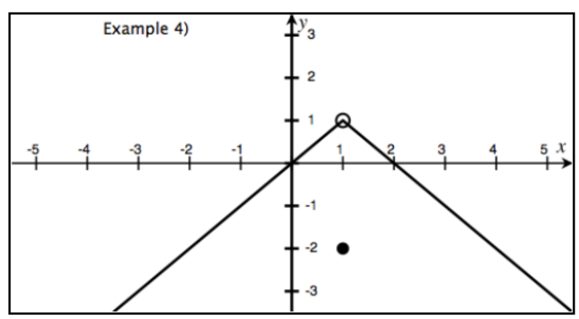


a.) $\lim\limits_{x \to 1^-} f(x) =$ _____ **b.)** $\lim\limits_{x \to 1^+} f(x) =$ _____

c.) $\lim\limits_{x \to 1} f(x) =$ _____ **d.)** $f(1) =$ _____



Scan the QR Code above to watch a video covering Examples 1-4.

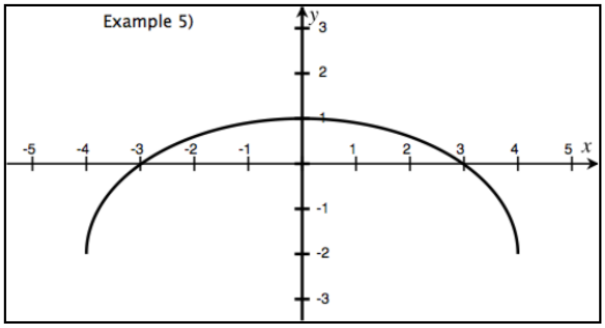


a.) $\lim\limits_{x \to 1^-} f(x) =$ _____ **b.)** $\lim\limits_{x \to 1^+} f(x) =$ _____

c.) $\lim\limits_{x \to 1} f(x) =$ _____ **d.)** $f(1) =$ _____

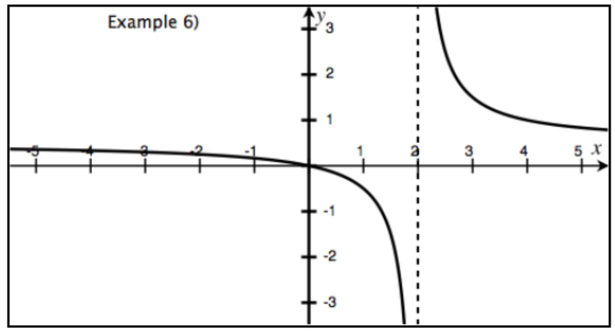
a.) $\lim\limits_{x \to 1^-} f(x) =$ _____ **b.)** $\lim\limits_{x \to 1^-} f(x) =$ _____

c.) $\lim\limits_{x \to 1} f(x) =$ _____ **d.)** $f(1) =$ _____



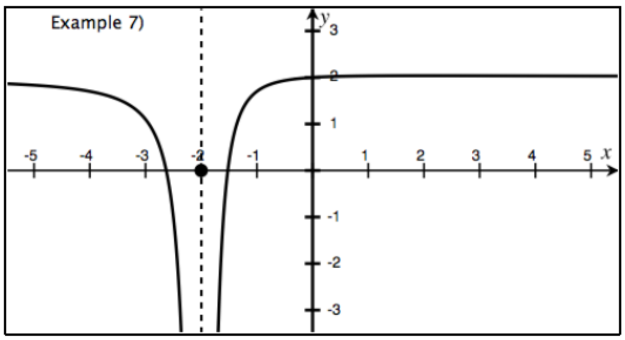
a.) $\lim_{x \to 0^-} f(x) =$ ______ **b.)** $\lim_{x \to 0^+} f(x) =$ ______

c.) $\lim_{x \to 0} f(x) =$ ______ **d.)** $f(0) =$ ______



a.) $\lim_{x \to 2^-} f(x) =$ ______ **b.)** $\lim_{x \to 2^+} f(x) =$ ______

c.) $\lim_{x \to 2} f(x) =$ ______ **d.)** $f(2) =$ ______



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Scan the QR Code above to watch a video covering Examples 5-7

a.) $\lim_{x \to -2^-} f(x) =$ ______ **b.)** $\lim_{x \to -2^+} f(x) =$ ______

c.) $\lim_{x \to -2} f(x) =$ ______ **d.)** $f(-2) =$ ______

Using Technology to Find Limits

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Scan the QR Code above to watch a video covering Examples 8-11

Use a graphing utility, when necessary, to find each limit.

Example 8: Find $\lim_{x \to 0} \dfrac{x}{\sqrt{x+1}-1}$.

Example 9: Find $\lim_{x \to 2} f(x)$ if $f(x) = \begin{cases} 1 & x \neq 2 \\ 0 & x = 2 \end{cases}$.

Example 10: Find $\lim_{x \to 0} \dfrac{|x|}{x}$.

Example 11: Find $\lim_{x \to 0} \left(\sin \dfrac{1}{x} \right)$.

Why Limits Fail to Exist

It is very important to understand the reasons why limits may fail to exist.

3 Reasons Why Limits Fail To Exist at $x = c$

1. $f(x)$ approaches a different number from the right side of c than it approaches from the left side side of c. This often called a "jump" discontinuity and happens often with piecewise function.
2. $f(x)$ either increases and/or decreases without bound as x approaches c. (This means the function approaches either ∞ or $-\infty$. See: Definition of Vertical Asymptote)
3. $f(x)$ oscillates between two **fixed** values as x approaches c.

Activity In Examples 8-11, you discovered that two of those limits do not exist.

Discussing with a partner, determine which of the three reasons from the box properly explain why those limits do not exist.

The limit in example number ________ does not exist because of reason number __________.

The limit in example number ________ does not exist because of reason number __________.

AP® CALCULUS MULTIPLE CHOICE QUESTION

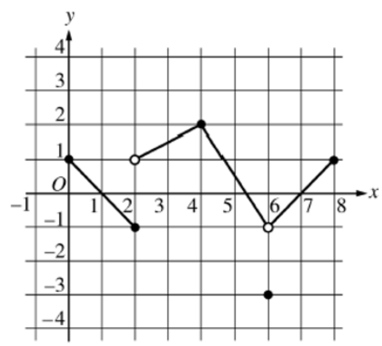
2014 BC MCQ #5

The figure to below shows the graph of the function *f*. Which of the following statements are true?

I. $\lim_{x\to 2^-} f(x) = f(2)$

II. $\lim_{x\to 6^-} f(x) = \lim_{x\to 6^+} f(x)$

III. $\lim_{x\to 6} f(x) = f(6)$



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(A) I only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II and III