LIM	AP CALCULUS BC	
3	Topic: 10.15	Representing Functions as Power Series
Learning Objectives LIM-8.G: Represent a given function as a power series.		

Example 1: Finding a Geometric Power Series Centered at 0.

Find a power series for $f(x) = \frac{4}{x+2}$, centered at 0 and state its interval of convergence.

Operations with Power Series

The following properties will make it very easy to write more complicated power series.

OPERATIONS WITH POWER SERIES

Let $f(x) = \sum_{n=0}^{\infty} a_n x^n$ and $g(x) = \sum_{n=0}^{\infty} b_n x^n$.

1. $f(kx) = \sum_{n=0}^{\infty} a_n (kx)^n = \sum_{n=0}^{\infty} a_n k^n x^n$

2. $f(x^N) = \sum_{n=0}^{\infty} a_n (x^N)^n = \sum_{n=0}^{\infty} a_n x^{nN}$

3. $f(x) \pm g(x) = \sum_{n=0}^{\infty} (a_n \pm b_n) x^n$

The operations described above can change the interval of convergence for the resulting series. For example, in the following addition, the interval of convergence for the sum is the *intersection* of the intervals of convergence of the two original series.

$$\underbrace{\sum_{n=0}^{\infty} x^n}_{(-1,1)} + \underbrace{\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n}_{\cap\ (-2,2)} = \underbrace{\sum_{n=0}^{\infty} \left(1 + \frac{1}{2^n}\right) x^n}_{=\ (-1,1)}$$

Example 2: Maclaurin Series for a Composite Function.

Find the Maclaurin series for each of the following functions. Be sure to list the first three non-zero terms and the general term.

a.) $f(x) = \sin x^2$

Example 2: Maclaurin Series for a Composite Function.

Find the Maclaurin series for each of the following functions. Be sure to list the first three non-zero terms and the general term.

b.) $f(x) = cos\sqrt{x}$

Example 3: Multiplication and Division of Power Series

Find the first three nonzero terms in the Maclaurin series for each of the given functions.

a. $f(x) = e^x \arctan x$ Video a

Example 4: AP Test Question

a.) What is the coefficient of x^6 in the Taylor Series for $f(x) = \frac{e^{3x^2}}{2}$ about $x = 0$