LIM		
3	Topic: 10.5	Harmonic Series and *p*-Series
Learning Objective LIM-7.A: Determine whether a series converges or diverges.		

This Topic will introduce one of the most common series, the ***p*-series**. Fortunately, it is perhaps the simplest type of series to determine convergence or divergence.

The *p*-Series

DEFINITION *p*-SERIES

A *p*-series is an infinite series of the form $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots$

where *p* is a positive number.

Scan the QR Code above to watch a video covering Example 1

Example 1: Recognizing a *p*-Series.

Which of the following series are *p*-series? State yes or no.

a.) $\sum_{k=1}^{\infty} k^{-0.8}$ **b.)** $\sum_{n=1}^{\infty} 2^{-n}$ **c.)** $\sum_{k=10}^{\infty} k^{-4}$

d.) $\sum_{n=1}^{\infty} n^2$ **e.)** $\sum_{k=1}^{\infty} \frac{1}{k}$

THEOREM 10.5-A: CONVERGENCE OF A *p*-SERIES

The *p*-series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots$

1. converges if $p > 1$, and

2. diverges if $0 < p \leq 1$.

Example 2: Convergent and Divergent *p*-Series.

Determine whether the following series converge or diverge.

a.) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[4]{k^3}}$ **b.)** $\sum_{n=4}^{\infty} \frac{1}{(n-1)^2}$ **c.)** $\sum_{k=2}^{\infty} \frac{1}{e^k}$

MATHEMATICS & HISTORY

PYTHAGORAS (c. 570 – c. 495 BC)
In addition to his countless contributions to geometry, Pythagoras and his students paid close attention to the development of music as an abstract science. This led to the discovery of the relationship between the tone and the length of a vibrating string. It was observed that the most beautiful musical harmonies corresponded to the simplest ratios of whole numbers. Later mathematicians developed this data into the harmonic series, where the terms in the series correspond to the nodes on a vibrating string that produce multiples of the fundamental frequency. For example, ½ is twice the fundamental frequency, 1/3 is three times the fundamental frequency, and so on.

Scan the QR Code above to watch a video covering Example 2

Example 3: **Convergent and Divergent *p*-Series.**

Determine the convergence or divergence of the following *p*-series.

$$\mathbf{1} + \frac{\mathbf{1}}{\sqrt[3]{\mathbf{4}}} + \frac{\mathbf{1}}{\sqrt[3]{\mathbf{9}}} + \frac{\mathbf{1}}{\sqrt[3]{\mathbf{16}}} + \frac{\mathbf{1}}{\sqrt[3]{\mathbf{25}}} + \cdots$$

SCAN ME

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The Harmonic Series

DEFINITION HARMONIC SERIES

A harmonic series is an infinite series of the form

$$\sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots$$

Note: A harmonic series is simply a *p*-series where the value of *p* is 1.

THEOREM 10.5-B: DIVERGENCE OF THE HARMONIC SERIES

The harmonic series

$$\sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots$$