LIM		
3	Topic: 10.6	Comparison Tests for Convergence
Learning Objective LIM-7.A: Determine whether a series converges or diverges.		

Direct Comparison Test

For the convergence tests that we have discussed so far, the terms of the series seem to be pretty simple and the series must have a special "look" to it in order for the convergence tests to be applied.
Any slight deviation from that "look" or characteristic, can make the convergence tests non-applicable.

Ex 1. $\sum_{n=0}^{\infty} \frac{1}{2^n}$ is geometric, but $\sum_{n=0}^{\infty} \frac{n}{2^n}$ is not.

Ex 2. $\sum_{n=1}^{\infty} \frac{1}{n^3}$ is a p-series, but $\sum_{n=0}^{\infty} \frac{1}{n^3+1}$ is not.

Our goal now becomes to investigate a way to **compare** a series with complicated terms with a simpler series whose convergence or divergence is known.

THEOREM: DIRECT COMPARISON TEST

Let $0 < a_n \le b_n$ for all *n.* (term by term comparison)

1. If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

2. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

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Example 1: Using the Direct Comparison Test

Determine the convergence or divergence of the following series.

$a.)\ \sum_{n=1}^{\infty} \frac{1}{2+3^n}$

$b.)\ \sum_{n=1}^{\infty} \frac{1}{2+\sqrt{n}}$

Remember that both parts of the Direct Comparison Test require that $0 < a_n \le b_n$.
Informally, the test says the following about the two series with nonnegative terms

1. **If the "larger" series converges, the "smaller" series must also converge.**
2. **If the "smaller" series diverges, then the "larger" series must also diverge.**

Limit Comparison Test

Often a series closely resembles a p-series or a geometric series, yet one **cannot** establish the term-by-term comparison necessary to apply the Direct Comparison Test.
If this happens, it may be possible to apply a second type of test called the **Limit Comparison Test.**

THEOREM: LIMIT COMPARISON TEST

Suppose $a_n > 0, b_n > 0$, and

$\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = L$ where L is ***finite*** *and* ***positive***. Then the two series $\sum a_n$ and $\sum b_n$ either both converge, or both diverge.

For the **Limit Comparison Test**

If $L = 0$ & $\sum_{n=1}^{\infty} b_n$ converges $\Rightarrow \sum_{n=1}^{\infty} a_n$ converges;

If $L = \infty$ & $\sum_{n=1}^{\infty} a_n$ converges $\Rightarrow \sum_{n=1}^{\infty} b_n$ converges.

NOTE: In the above cases, I would use a different test

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Example 2: Using the Limit Comparison Test

Show that the following general harmonic series diverges.

$$\sum_{n=1}^{\infty} \frac{1}{an+b}, a > 0, b > 0$$

Example 3: Using the Limit Comparison Test.

Determine the convergence or divergence of the following series.

a. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$

b. $\sum_{n=1}^{\infty} \frac{n2^n}{4n^3+1}$

Helpful examples in choosing an appropriate p-series when using the Limit Comparison Test

General Series	Comparison Series	Conclusion
$\sum_{n=1}^{\infty} \frac{1}{3n^2-4n+5}$	$\sum_{n=1}^{\infty} \frac{1}{n^2}$	Both series converge.
$\sum_{n=1}^{\infty} \frac{1}{\sqrt{3n-2}}$	$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$	Both series diverge.
$\sum_{n=1}^{\infty} \frac{n^2-10}{4n^5+n^3}$	$\sum_{n=1}^{\infty} \frac{n^2}{n^5} = \sum_{n=1}^{\infty} \frac{1}{n^3}$	Both series converge.