LIM	AP CALCULUS BC	
3	Topic: 10.7	Alternating Series Test for Convergence
Learning Objective LIM-7.A: Determine whether a series converges or diverges.		

Alternating Series

Up to now, most series we have dealt with had positive terms. In this section and beyond, we'll study series that contain both positive and negative terms. The simplest of these series is called an **alternating series** whose terms alternate signs.

DEFINITION: ALTERNATING SERIES

An alternating series is a series either of the form

$$\sum_{n=1}^{\infty}(-1)^{n+1}a_n = a_1 - a_2 + a_3 - a_4 + \cdots \quad \text{or}$$

$$\sum_{n=1}^{\infty}(-1)^{n}a_n = -a_1 + a_2 - a_3 + a_4 - \cdots \text{ where } a_n > 0 \text{ for all integers } n \ge 1.$$

MATHEMATICS & HISTORY

GOTTFRIED WILHELM LEIBNIZ
(1646-1716)
Leibniz is a German mathematician and philosopher who is credited as one of the two main discoverers of calculus having worked independ of Isaac Newton. In addition to developing the common notation for derivatives, dy/dx, that we still use today, Leibniz developed the Alternating Series test which is sometimes called the Leibniz Test or Leibniz Criterion.

THEOREM 10.7: ALTERNATING SERIES TEST

Let $a_n > 0$. The alternating series $\sum_{n=1}^{\infty}(-1)^{n}a_n$ and $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$

converge if the following two conditions are met

1. $\lim_{n\to\infty} a_n = 0$ **2.** If $a_{n+1} \le a_n$ for all n

Note: Another way to state Criterion #2 above is that the a_n values are **nonincreasing**.

Example 1: Using the Alternating Series Test

Determine the convergence or divergence of the following series.

a.) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$

b.) $\sum_{n=1}^{\infty}(-1)^{n}\frac{n}{n+1}$

SCAN ME

Scan the QR Code above to watch a video covering Example 1

VERIFYING NONINCREASING BEHAVIOR FOR A SEQUENCE

1. Use the **Algebraic Difference** test to show that $a_{n+1} - a_n \leq 0$ for all $n \geq 1$.

2. Use the **Algebraic Ratio** test to show that $\dfrac{a_{n+1}}{a_n} \leq 1$ for all $n \geq 1$.

3. Use the **derivative** of the related function, $f(x)$, for which $a_n = f(n)$ for all n, defined for $x > 0$ and show that $f'(x) \leq 0$ for all $x \geq 0$.

Example 2: Showing and Alternating Series Converges

Show that the following alternating series convergences.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!}$$

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