LIM	AP CALCULUS BC		
3	Topic: 10.9	Determining Absolute or Conditional Convergence	
Learning Objective LIM-7.A: Determine whether a series converges or diverges.			

Absolute and Conditional Convergence

The concepts of **absolute** and **conditional** convergence are used to determine the convergence (or divergence) of a series in which the terms are sometimes positive and sometimes negative – but not necessarily alternating.
We begin this analysis of series behavior with the following theorem.

THEOREM 10.9: ABSOLUTE CONVERGENCE

If the series $\sum |a_n|$ converges, then the series $\sum a_n$ also converges.

Note: The converse of the above theorem is not true. For instance, the **alternating harmonic series**

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$

converges by the Alternating Series Test. Yet the harmonic series diverges. This type of convergence is called **conditional.**

DEFINITIONS OF ABSOLUTE AND CONDITIONAL CONVERGENCE

1. $\sum a_n$ is **absolutely convergent** if $\sum |a_n|$ converges.
2. $\sum a_n$ is **conditionally convergent** if $\sum a_n$ converges but $\sum |a_n|$ diverges.

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Example 1: Absolute and Conditional Convergence

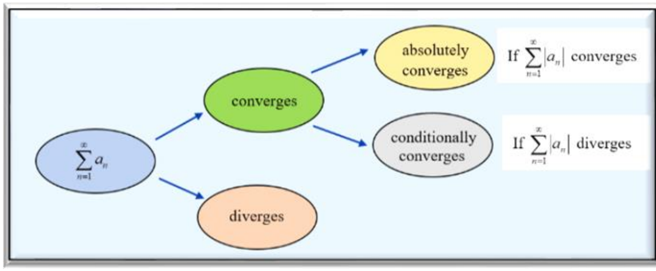
Determine whether each of the series is absolutely convergent, conditionally convergent, or divergent.

a. $$\sum_{n=0}^{\infty} \frac{(-1)^n n!}{2^n} = \frac{0!}{2^0} - \frac{1!}{2^1} + \frac{2!}{2^2} - \frac{3!}{2^3} + \cdots$$

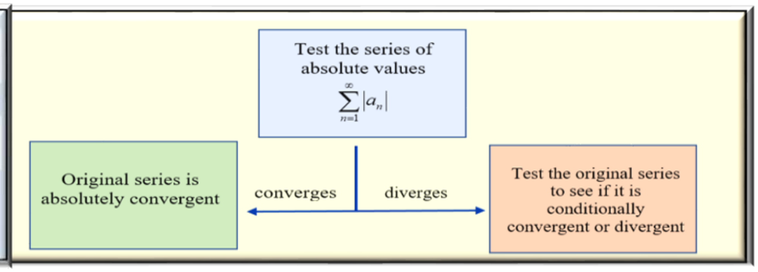
b. $$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} = -\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \cdots$$

The following pair of flowcharts may help you when trying to determine whether a series is absolutely convergent, conditionally convergent, or divergent.

Flowchart 1 – Absolute vs Conditional Convergence



Flowchart 2 – Absolute vs Conditional



Convergence

Example 2: Absolute and Conditional Convergence

Determine whether each of the series is absolutely convergent, conditionally convergent, or divergent.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n^2+1}$

b. $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}2^k}{k^2}$

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Rearrangement of Series

A finite sum such as (1 + 3 – 2 + 5 – 4) can be rearranged without changing the value of the sum. This, however, is not necessarily true of an infinite series—it depends on whether the series is absolutely convergent (where *every* rearrangement has the same sum) or conditionally convergent.

Example 3: Rearrangement of a Series

Mathematicians have shown that the alternating harmonic series $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{n}$ converges to $\ln 2$.

That is to say $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{n} = \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots = \ln 2$.

Rearrange the series to produce a different sum.

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