Pre-Calculus Notes Name: ___________________

Section 2.5 – Zeroes of Polynomial Functions

Rational Zeros (Roots) Theorem:

Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0$ (where $a_0 \neq 0$) be a polynomial function in standard form that has integral coefficients. THEN if the nonzero rational number $\frac{p}{q}$ in lowest terms is a zero of $p(x)$, p must be a factor of the constant term a_0 AND q must be a factor of the leading coefficient a_n.

Example 1: Determine the **possible** rational zeros of each polynomial.

a. $f(x) = x^4 - 7x^3 - 3x^2 + 2x + 12$	b. $g(x) = 6x^4 - 3x^3 + x^2 - 10x + 15$

What kind of polynomial equation(s) can I solve easily? ________________________

Example 2: Determine the EXACT VALUES of the zeroes of each polynomial. Use your calculator to get started.

a. $p(x) = 7x^3 + 18x^2 - 97x - 60$	b. $f(x) = 22x^4 + 65x^3 - 20x^2 - 45x + 18$

Fundamental Theorem of Algebra:

Every polynomial function of positive degree with complex coefficients has at least one complex zero.

NOTE: The zero may be a real number since ANY real number r can be expressed as the complex number $r+0i$.

Number of Roots Theorem:

If $f(x)$ is a polynomial of degree $n\geq 1$ with complex coefficients, then $f(x)=0$ has exactly n roots, where roots are counted according to their multiplicity.

Conjugate Pair Theorem:

If $f(x)=0$ is a polynomial equation real coefficients, then when $a+bi$ is a root, $a-bi$ is also a root. If $f(x)=0$ is a polynomial equation rational coefficients, then when $m+\sqrt{n}$ is a root, $m-\sqrt{n}$ is also a root.

Example 3: Determine the zeros of each polynomial.

a. $f(x)=x^4-x^3+x^2-3x-6$	b. $g(x)=x^5+5x^4-8x^3-40x^2$
c. $f(x)=9x^4+131x^3+183x^2+9x-52$	d. $g(x)=2x^4+11x^3+2x^2-65x-100$

Example 4: Determine the remaining zeros of the polynomial given one of the zeros. Explain how you arrived at your answer.

$f(x) = x^4 - 13x^3 + 61x^2 - 127x + 78; \quad 3 + 2i$

Example 5: Use your calculator to answer the following questions about the given polynomial function.

$f(x) = x^4 - 7x^3 - 46x^2 + 14x + 88$

Possible Rational Roots: ____

Actual Rational Roots: ______________

# of Real Roots: ______________

# of Irrational Roots: ______________

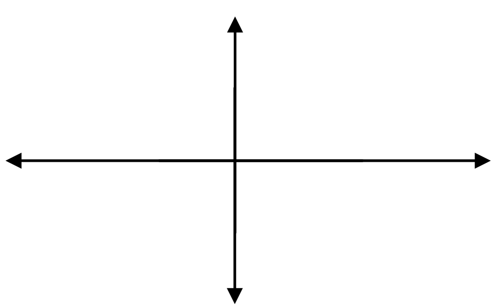
Irrational Roots: ____________________ (Use the zero feature on your graphing calculator to estimate.)

# of Imaginary Roots: ______________

Imaginary Roots: ______________

Example 6: Sketch a possible graph with the following conditions.

a. A fourth degree polynomial with a positive leading coefficient, two distinct negative real zeros greater than -5, and one positive real zero less than 4 with a multiplicity of 2.



b. A fifth degree polynomial with a negative leading coefficient, two distinct negative real zeros greater than –4 (one with multiplicity 3), and one positive real zero less than 3.