

## Section 2.5 – Concavity

### Definitions.

1. A function  $f(x)$  is called *increasing* if its graph \_\_\_\_\_ from left to right. It is called *decreasing* if its graph \_\_\_\_\_ from left to right.
2. A function  $f(x)$  is called *concave up* if its average rate of change increases from left to right.
3. A function  $f(x)$  is called *concave down* if its average rate of change decreases from left to right.

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Describe the shape of the graph of a function  $f(x)$  that is concave up:

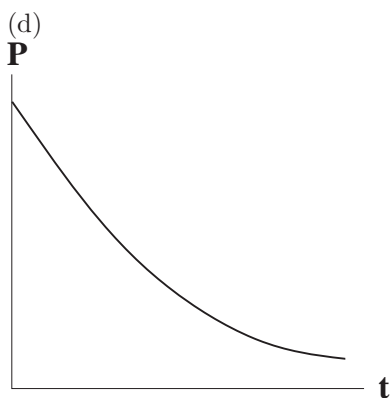
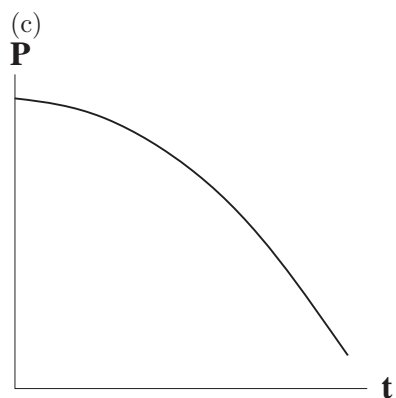
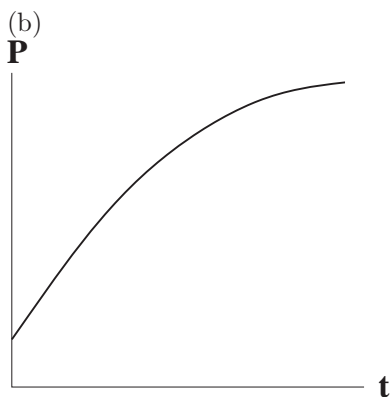
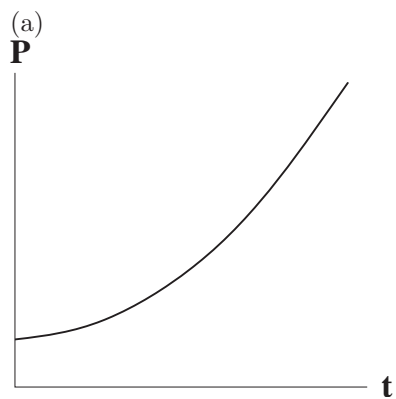
Describe the shape of the graph of a function  $f(x)$  that is concave down:

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**Example.** Read the following description of a function. Then, decide whether the function is increasing or decreasing. What does the scenario tell you about the concavity of the graph modeling it?

“When a new product is introduced, the number of people who use the product increases slowly at first, and then the rate of increase is faster (as more and more people learn about the product). Eventually, the rate of increase slows down again (when most people who are interested in the product are already using it).”

**Example.** Consider the following graphs of population,  $P$ , as a function of time,  $t$ .



### Descriptions

(a)  $P$  is \_\_\_\_\_, and the rate of change of  $P$  is \_\_\_\_\_.

(b)  $P$  is \_\_\_\_\_, and the rate of change of  $P$  is \_\_\_\_\_.

(c)  $P$  is \_\_\_\_\_, and the rate of change of  $P$  is \_\_\_\_\_.

(d)  $P$  is \_\_\_\_\_, and the rate of change of  $P$  is \_\_\_\_\_.

2. Decide whether each of the following functions are concave up, concave down, or neither.

|        |   |   |   |    |    |
|--------|---|---|---|----|----|
| $x$    | 0 | 1 | 2 | 3  | 4  |
| $f(x)$ | 1 | 3 | 6 | 10 | 20 |

|        |    |   |   |   |   |
|--------|----|---|---|---|---|
| $x$    | 0  | 1 | 2 | 3 | 4 |
| $g(x)$ | 10 | 9 | 7 | 4 | 0 |

$$p(x) = 3x + 1$$

