Notes 2.5 Zeros of Polynomial Functions, Day 2

What you should know already

1. How to use the Fundamental Theorem of Algebra to determine the number of zeros of a polynomial function.

2. How to find *possible* rational zeros of polynomial functions

3. How to test each possible rational zero

4. How to find conjugate pairs of complex zeros

What you will learn today

How to write a polynomial function as the product of linear factors and list *all* of its zeros and why a graph cannot be used to locate complex zeros.

EXAMPLE 1:

$f(x) = x^4 + 5x^2 - 36$

What is Descartes's Rule of Signs?

Descartes's Rule of Signs:

1. **The number of _____ of a function *f* is either equal to the number of sign changes of $f(x)$ or less than by an _____ integer.**
2. **The number of _____ of a function *f* is either equal to the number of sign changes of $f(-x)$ or less than by an _____ integer.**
3. **When using Descartes's Rule of Signs, a zero of multiplicity k should be counted as _____ zeros.**

How to use Descartes's Rule of Signs

EXAMPLE 2: $f(x) = x^3 - 3x + 2$ has 3 real zeros. How many are positive? How many are negative?

Upper and Lower Bound Rules

Let $f(x)$ be a polynomial with a positive leading coefficient. Suppose $f(x)$ is divided by $(x - c)$, using synthetic division,

- If $c > 0$ and each number in the last row is either positive or zero, then c is an upper bound for the real zeros
- If $c < 0$ and the numbers alternate from positive to negative (zero counts as either), then c is a lower bound for the real zeros

How to use Descartes's Rule of Signs and the Upper and Lower Bound Rules to find zeros of polynomials

EXAMPLE 3: $f(x) = 6x^3 - 4x^2 + 3x - 2$

a) Find the number of sign changes in $f(x)$

b) Find the number of sign changes in $f(-x)$

c) Make a **P**ositive, **N**egative, **I**maginary chart (**PNI**)

d) Find all the *possible* rational zeros of $f(x)$

e) Find an upper and lower bound to help eliminate possible rational zeros

f) Find any rational zeros

EXAMPLE 4: Use P(x)= $2x^7 - x^6 - 42x^3 + 21x^2 + 40x - 20$ to answer the following questions. Be sure to label your work clearly.

a) Use Descarte's Rule to find the possible numbers of positive and negative real roots of P(x). Make a PNI chart.

b) Use the Rational Root Theorem to list <u>all possible</u> rational roots of P(x).

c) Using synthetic division, find the zeros and possible upper and lower bounds.

d) Give the prime factorization of P(x) into linear **and irreducible** quadratic factors over the real number system.

e) Give the prime factorization of P(x) into **linear factors** of complex coefficients.

f) Solve P(x) = 0 over the complex field ***C. Find all real and complex zeros.***

g) Sketch a graph of the polynomial function.