AP CALCULUS NOTES 3.1

Slopes of Secant and Tangent Lines

If $P(x_0, y_0)$ and $Q(x_1, y_1)$ are distinct points on a curve, then the **secant line** connecting P and Q has slope

$$m_{\text{sec}} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}.$$

The **slope of the tangent line** at $x = x_0$ is

$$m_{\text{tan}} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}. \quad \text{Method \#1}$$

$$m_{tan} = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} \quad \text{Method \#2}$$

Average and Instantaneous Rates of Change

If $y = f(x)$, the **average rate of change** of y with respect to x over the interval $[x_0, x_1]$ is the slope m_{sec} of the secant line joining the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$ on the graph of f; or

$$m_{\text{sec}} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}.$$

If $y = f(x)$, the **instantaneous rate of change** of y with respect to x at the point x_0, is the slope m_{tan} of the tangent line to the graph of f at the point x_0; or

$$m_{\text{tan}} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}.$$

$$m_{tan} = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

Average and Instantaneous Velocity

If an objects initial position is $s_0 = f(t_0)$ at time t_0 and its position is $s_1 = f(t_1)$ at time t_1, where $t_1 > t_{0,}$ then the **average velocity** of the object during that time interval is

$$v_{ave} = \frac{s_1 - s_0}{t_1 - t_0} = \frac{f(t_1) - f(t_0)}{t_1 - t_0}.$$

For an object moving in a positive direction along a coordinate line, the average velocity of the object between time t_0 *and* t_1 *is represented geometrically by the slope of the secant line connecting* (t_0, s_0) and (t_1, s_1) *on the position versus time curve.*

The objects **instantaneous velocity** at $t = t_0$ is

$$v_{inst} = \lim_{t_1 \to t_0} v_{ave} = \lim_{t_1 \to t_0} \frac{f(t_1) - f(t_0)}{t_1 - t_0}.$$

For an object moving in a positive direction along a coordinate line, the instantaneous velocity of the object at time t_0 *is represented geometrically by the slope of the tangent line at* (t_0, s_0) *on the position versus time curve.*