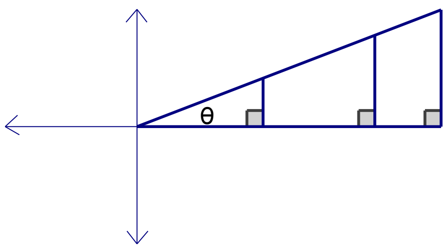
Pre-Calculus Notes Name: ____________________

Sections 4.2 and 4.4 Meshed Together…

PART ONE: We looked at trigonometric functions for angles measuring between 0° and 90° when we focused on the acute angles inside of a right triangle. We can also assign trig. values to ANY angle measure, including angles greater than 90°. But need to understand **reference angles** and **reference triangles** first.

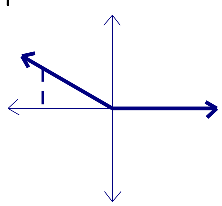
First, let's review an important geometry concept…

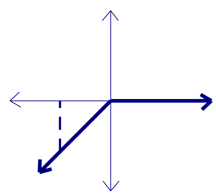
Will the angle θ have the same trig values REGARDLESS of the triangle used? ___________ **Why?**

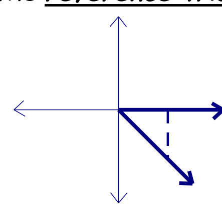


Second, let's look at reference angles and reference triangles.

A ***reference angle*** is the ACUTE angle (always positive) formed by the TERMINAL side of any angle in standard position and the nearest portion of the x-axis. The triangle formed is the ***reference triangle***.







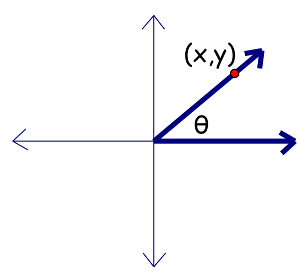
Third, let's start the associating the trig values with x, y, and r, as well as adj, opp, and hyp.

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$, since $x^2 + y^2 = r^2$. Then…

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{1}{\sin\theta} =$$

$$\cos\theta = \frac{x}{r} \quad \text{and} \quad \sec\theta = \frac{1}{\cos\theta} =$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{1}{\tan\theta} =$$



Example 1: Find the 6 trigonometric function values of the angle with a point $(3, 4)$ on the terminal side of the angle, θ.

$x =$ ______ $y =$ ______ $r =$ ______

$\sin\theta =$ ________ $\cos\theta =$ ________ $\tan\theta =$ ________

$\csc\theta =$ ________ $\sec\theta =$ ________ $\cot\theta =$ ________

Example 2: Find the 6 trigonometric function values of the angle with a point $(-5,12)$ on the terminal side of the angle, θ.

$x =$ ______ $y =$ ______ $r =$ ______

$\sin\theta =$ ________ $\cos\theta =$ ________ $\tan\theta =$ ________

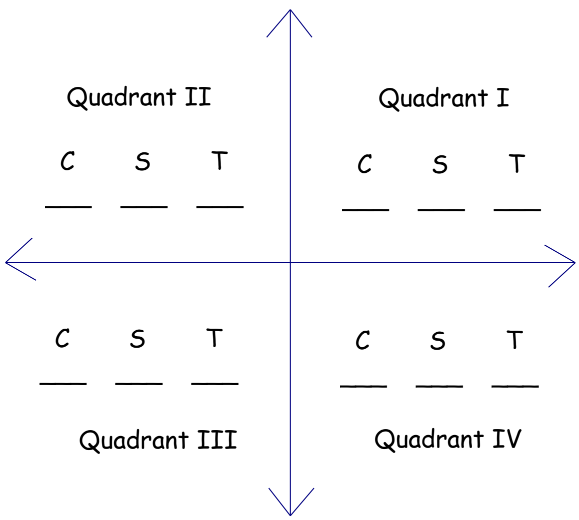
$\csc\theta =$ ________ $\sec\theta =$ ________ $\cot\theta =$ ________

Now let's look at determining the sign (positive or negative) of a function by looking at the quadrant in which the angle terminates.

The signs of the trigonometric functions in the four quadrants can be determined easily from the definitions of the functions. For instance, because $\cos\theta = \frac{x}{r}$, it follows that $\cos\theta$ is positive wherever $x > 0$, which is in Quadrants I and IV. (Remember, r is always positive.)

Where will $\sin\theta$ be positive? ____

Where will $\tan\theta$ be positive?____



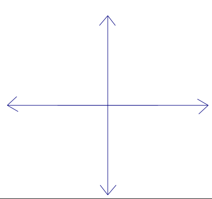
So… what is a trick to help me remember?

"All Stupid Teachers Cheat"

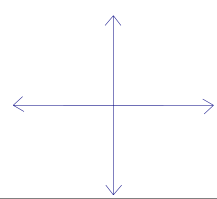
can help you identify which of the main trig. functions ($\sin$, $\cos$, and $\tan$) are positive for each quadrant. Go counter-clockwise and then just remember that the trig. functions reciprocal will have the same sign (so $\csc$ is positive in all the quadrants where $\sin$ is positive).

Example 3: State the quadrant(s) in which θ terminates based on the given information.

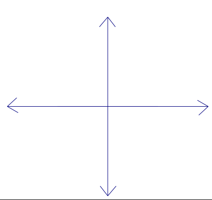
a. $\cos\theta > 0$



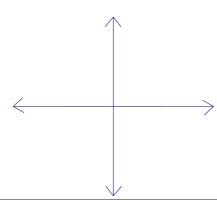
b. $\sin\theta < 0$, $\cos\theta < 0$



c. $\sec\theta < 0$, $\tan\theta < 0$

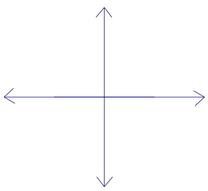


d. $\csc\theta > 0$, $\cot\theta < 0$



Example 4: Given $\tan\theta = -\frac{5}{4}$ and $\cos\theta > 0$, find the exact values of the other 5 functions.

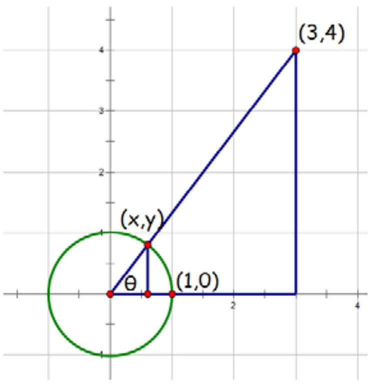
$x =$ ______ $y =$ ______ $r =$ ______



$\sin\theta =$ ________ $\cos\theta =$ ________ $\tan\theta =$ ________

$\csc\theta =$ ________ $\sec\theta =$ ________ $\cot\theta =$ ________

We will be focusing on the Unit Circle more and more over the next few days. What is a Unit Circle? Well, it is a circle that is centered at the origin and that has a radius of one unit. **What effect will that have?**



What are the coordinates for x and y? ________________________

What is the cosine of θ? ________________________

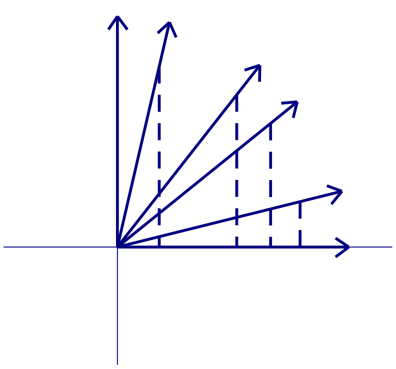
What is the sine of θ? ________________________

Hence, in the UNIT CIRCLE…

- **the cosine of an angle is always** ________________________
- **AND the sine of an angle is always** ______________________.

Quadrantal Angles –

Now, we will focus on angles in standard position that terminate on an axis, also known as **quadrantal angles.**



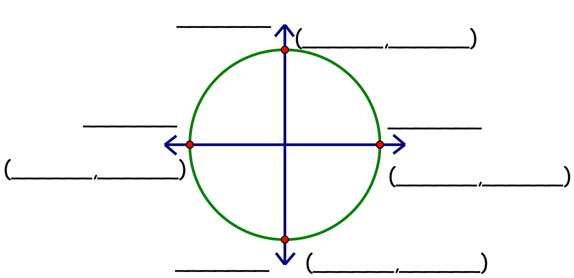
What happens as the terminal side of the angle approaches the x-axis?

What happens when the terminal side is ON the x-axis?

What happens as the terminal side of the angle approaches the y-axis?

What happens when the terminal side is ON the y-axis?

Armed with this knowledge, let's figure out the trig function values for any angle whose terminal side lies on an axis, also known as a **quadrantal angle**.



REMEMBER…

$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ and $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$