

Pre-Calculus Notes

Name: \_\_\_\_\_

Section 4.3 – The 6 Trigonometric Functions

**PART ONE:** Definitions Using Right Triangles: **SOH-CAH-TOA**

$$\sin \theta = \frac{\textit{opposite}}{\textit{hypotenuse}}$$

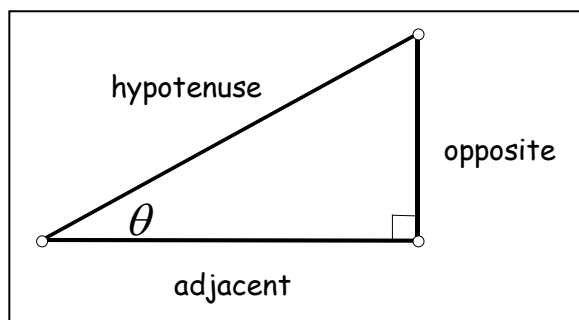
$$\csc \theta = \frac{\textit{hypotenuse}}{\textit{opposite}}$$

$$\cos \theta = \frac{\textit{adjacent}}{\textit{hypotenuse}}$$

$$\sec \theta = \frac{\textit{hypotenuse}}{\textit{adjacent}}$$

$$\tan \theta = \frac{\textit{opposite}}{\textit{adjacent}}$$

$$\cot \theta = \frac{\textit{adjacent}}{\textit{opposite}}$$



Example 1: Find the values of the six trigonometric functions of  $\theta$ .

$\sin \theta = \underline{\hspace{2cm}}$      $\cos \theta = \underline{\hspace{2cm}}$      $\tan \theta = \underline{\hspace{2cm}}$   
 $\csc \theta = \underline{\hspace{2cm}}$      $\sec \theta = \underline{\hspace{2cm}}$      $\cot \theta = \underline{\hspace{2cm}}$

Example 2: Find the 6 trig. values of the angle shown. Give exact values, simplified.

$\sin \theta = \underline{\hspace{2cm}}$      $\cos \theta = \underline{\hspace{2cm}}$      $\tan \theta = \underline{\hspace{2cm}}$   
 $\csc \theta = \underline{\hspace{2cm}}$      $\sec \theta = \underline{\hspace{2cm}}$      $\cot \theta = \underline{\hspace{2cm}}$

Example 3: Sketch a right triangle corresponding to the given trig. function and its value. Find the third side and the other trig. values.

$$\sin \theta = \underline{\hspace{2cm}}$$
     $\cos \theta = \frac{3}{5}$      $\tan \theta = \underline{\hspace{2cm}}$   

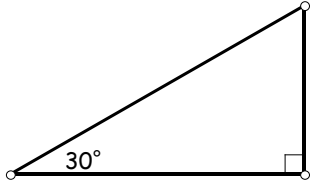
$$\csc \theta = \underline{\hspace{2cm}}$$
     $\sec \theta = \underline{\hspace{2cm}}$      $\cot \theta = \underline{\hspace{2cm}}$

You will need to have the following trig value **MEMORIZED** very "SOON".

Example 4: Find the values of the six trigonometric functions of  $\theta$  in a  $45^\circ - 45^\circ - 90^\circ$  triangle.

$\sin 45^\circ = \underline{\hspace{2cm}}$      $\cos 45^\circ = \underline{\hspace{2cm}}$      $\tan 45^\circ = \underline{\hspace{2cm}}$   
 $\csc 45^\circ = \underline{\hspace{2cm}}$      $\sec 45^\circ = \underline{\hspace{2cm}}$      $\cot 45^\circ = \underline{\hspace{2cm}}$

Example 5: Find the values of the six trigonometric functions of  $\theta$  in a  $30^\circ - 60^\circ - 90^\circ$  triangle.



$\sin 30^\circ = \underline{\hspace{2cm}}$      $\cos 30^\circ = \underline{\hspace{2cm}}$      $\tan 30^\circ = \underline{\hspace{2cm}}$   
 $\csc 30^\circ = \underline{\hspace{2cm}}$      $\sec 30^\circ = \underline{\hspace{2cm}}$      $\cot 30^\circ = \underline{\hspace{2cm}}$   
 $\sin 60^\circ = \underline{\hspace{2cm}}$      $\cos 60^\circ = \underline{\hspace{2cm}}$      $\tan 60^\circ = \underline{\hspace{2cm}}$   
 $\csc 60^\circ = \underline{\hspace{2cm}}$      $\sec 60^\circ = \underline{\hspace{2cm}}$      $\cot 60^\circ = \underline{\hspace{2cm}}$

Could we summarize the above information into a single chart? Yes!!!!!!!!!!!!!!

	$\frac{\pi}{6}$ or $30^\circ$	$\frac{\pi}{4}$ or $45^\circ$	$\frac{\pi}{3}$ or $60^\circ$
$\cos \theta$			
$\sin \theta$			
$\tan \theta$			

Example 6: Fill in the blanks with the angle measure or function value.

Function	$\theta$ in degrees	$\theta$ in radians	Value
sec			$\sqrt{2}$
tan	$45^\circ$		
cos		$\frac{\pi}{3}$	
csc			2

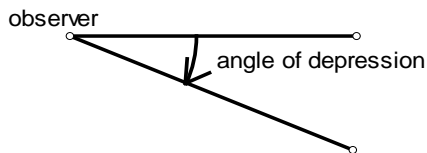
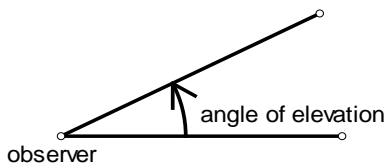
Now let's do these on the CALCULATOR...

Example 2: Calculate to 4 decimal places.

a. $\sin 25^\circ$	b. $\cos 32.4^\circ$	c. $\tan 40^\circ 15' 10''$
d. $\csc 85^\circ$	e. $\sec 75^\circ 30'$	f. $\cot 56.27^\circ$

### Applications - Solving Right Triangles

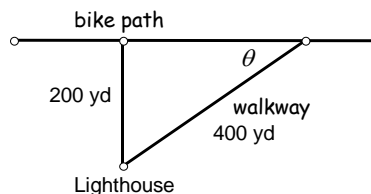
Do you remember angle of elevation and angle of depression?



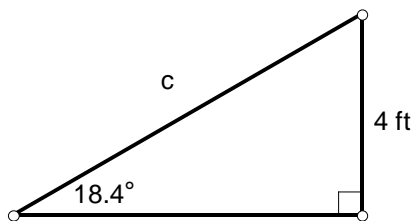
Example 3:

A surveyor is standing 115 feet from the base of the Washington Monument. The surveyor measures the angle of elevation to the top of the monument as  $78.3^\circ$ . How tall is the Washington Monument?

An historic lighthouse is 200 yards from a bike path along the lake. A walkway to the lighthouse is 400 yards long. Find the acute angle between the bike path and the walkway.

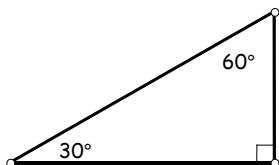


Example 5: Solve for  $c$ .



**PART TWO:** Let's talk now about the topic of "CO-FUNCTIONS".

Think about the  $30^\circ - 60^\circ - 90^\circ$  triangle.



$\sin 30^\circ =$ _____	↔	$\cos 60^\circ =$ _____
$\tan 30^\circ =$ _____	↔	$\cot 60^\circ =$ _____
$\sec 30^\circ =$ _____	↔	$\csc 60^\circ =$ _____

WHY are those values equal? \_\_\_\_\_

WHEN will those function values be equal? \_\_\_\_\_

In general, it can be shown from the right triangle definitions that **co-functions of complementary angles are equal**. That is, if  $\theta$  is an acute angle, the following relationships are true:

$$\sin \theta = \cos(90^\circ - \theta) \text{ or } \cos\left(\frac{\pi}{2} - \theta\right)$$

$$\tan \theta = \cot(90^\circ - \theta) \text{ or } \cot\left(\frac{\pi}{2} - \theta\right)$$

$$\sec \theta = \csc(90^\circ - \theta) \text{ or } \csc\left(\frac{\pi}{2} - \theta\right)$$

So, let's try this again. What functions are **CO-FUNCTIONS**?

\_\_\_\_\_ AND \_\_\_\_\_      \_\_\_\_\_ AND \_\_\_\_\_      \_\_\_\_\_ AND \_\_\_\_\_

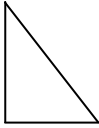
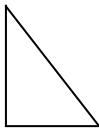
WHEN will the co-function values be equal? \_\_\_\_\_

We also need to start looking at some of the **TRIG. IDENTITIES**.

Look at the following and discuss **WHY** they are true.

<b>Fundamental Identities:</b>	
<b>Reciprocal Identities:</b>	<b>Quotient Identities:</b>
$\sin \theta = \frac{1}{\csc \theta} ; \quad \csc \theta = \frac{1}{\sin \theta}$ $\cos \theta = \frac{1}{\sec \theta} ; \quad \sec \theta = \frac{1}{\cos \theta}$ $\tan \theta = \frac{1}{\cot \theta} ; \quad \cot \theta = \frac{1}{\tan \theta}$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\cot \theta = \frac{\cos \theta}{\sin \theta}$

**Example 1:** Use **CO-FUNCTIONS** and **TRIG IDENTITIES** to find the other values.

If $\sec \theta = 5$		and $\tan \alpha = 2\sqrt{6}$ ,		then find the following...
a. $\cos \theta$		b. $\cot \alpha$		
c. $\csc(90^\circ - \theta)$		d. $\cot(90^\circ - \alpha)$		
e. $\csc \theta$		f. $\tan(90^\circ - \alpha)$		