AP CALCULUS - 5.2

ANTIDERIVATIVES AND INTEGRALS

A function F is called an **<u>antiderivative</u>** of a function f on a given interval if $F'(x) = f(x)$ for all x in that interval.

The process of finding antiderivatives is called **<u>antidifferentiation</u>** or **<u>integration</u>**.

If some function F exists such that $\frac{d}{dx}\left[F(x)\right] = f(x)$ then the function**s** of the form $F(x)+C$ are antiderivative**s** of $f(x)$.

Mathematically: $\int f(x)dx = F(x)+C$ which is read,

"the **indefinite integral** of $f(x)$ equals $F(x)$ plus C."

Indefinite Integral means "generic" function

$\int$ is the **integral sign**

$f(x)$ is the **integrand**

C is the **constant of integration**

The dx symbol identifies the **independent variable**

Formulas:

<u>DIFFERENTIATION</u>	**<u>INTEGRATION</u>**
$\frac{d}{dx}\left[x\right]=1$	$\int 1\,dx = x+C$
$\frac{d}{dx}\left[\frac{x^{r+1}}{r+1}\right]=x^r \quad (r \neq -1)$	$\int x^r\,dx = \frac{x^{r+1}}{r+1}+C$
$\frac{d}{dx}\left[\sin x\right]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}\left[-\cos x\right]=\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}\left[\tan x\right]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}\left[-\cot x\right]=\csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$
$\frac{d}{dx}\left[\sec x\right]=\sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}\left[-\csc x\right]=\csc x \cot x$	$\int \csc x \cot x\,dx = -\csc x + C$

ANTIDERIVATIVES AND INTEGRALS

Formulas:

DIFFERENTIATION	**INTEGRATION**
$\frac{d}{dx}\left[e^x\right]=e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}\left[e^{-x}\right]=-e^{-x}$	$\int e^{-x} dx = -e^{-x} + C$
$\frac{d}{dx}\left[\frac{1}{\ln b}\cdot b^x\right]=b^x$	$\int b^x dx = \frac{b^x}{\ln b}+C, \quad b>0, b\neq 1$
$\frac{d}{dx}\left[\ln\lvert x\rvert\right]=\frac{1}{x}$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$
$\frac{d}{dx}\left[\tan^{-1} x\right]=\frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$
$\frac{d}{dx}\left[\sin^{-1} x\right]=\frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$
$\frac{d}{dx}\left[\sec^{-1}\lvert x\rvert\right]=\frac{1}{x\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1}\lvert x\rvert + C$

Properties of the Indefinite Integral

- Differentiating an antiderivative of $f(x)$ results in $f(x)$.

$$\frac{d}{dx}\left[\int f(x)\,dx\right]=f(x)$$

- A constant factor can be moved through an integral sign.

$$\int c\cdot f(x)\,dx = c\int f(x)\,dx$$

- The antiderivative of a sum or difference is the sum or difference of the antiderivatives.

$$\int \left[f(x)\pm g(x)\right]dx = \int f(x)\,dx \pm \int g(x)\,dx$$