Topic: 7.3 Sketching Slope Fields and Topic 7.4 Reasoning Using Slope Fields

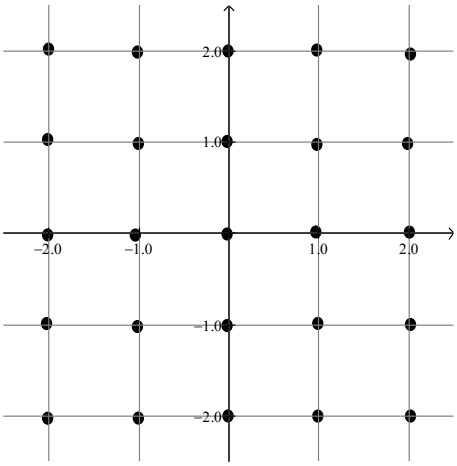
Learning Objective FUN-7.C: Estimate solutions to differential equations

Slope Fields

In a later lesson, we will learn ways to solve a differential equation analytically. However, doing so can be difficult or sometimes impossible. A graphical approach to solve a differential equation is by creating slope fields, which show the general shape of all solutions to a differential equation.

Consider a differential equation $y' = F(x, y)$ in terms of *x* and *y*. For every point (x, y) in its domain, y' determines the slope of the solution function at that point. If you draw a short line segment with the slope indicated at each point (mini tangent lines) on y', the **slope field (direction field)** will show the general shape of **all** the solution functions to that differential equation.

Example 1: Consider the following differential equation $\frac{dy}{dx} = \frac{-x}{2y}$.



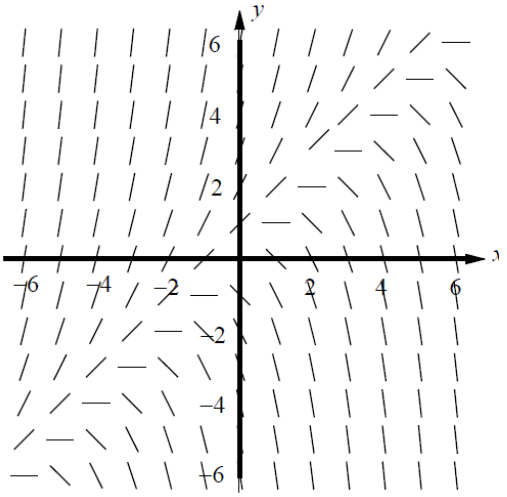
a. Sketch a slope field of the differential equation on the coordinate plane. Plug in (x, y) into the given differential equation and then draw a short line segment by estimating the appropriate slope.

NOTE: It's easiest to start where the derivative (slope) is equal to zero. Also, whenever the denominator of the differential equation equals 0, we do not want to draw a slope segment. This is because the derivative is not defined at these points. We cannot guarantee *why* the derivative is undefined though.

b. Sketch one possible solution curve to the differential equation that passes through $(0,1)$ on the coordinate plane to the right.

Example 2:

Shown to the right is a slope field for which of the following differential equations?



(A) $\frac{dy}{dx} = x + y$

(B) $\frac{dy}{dx} = x - y$

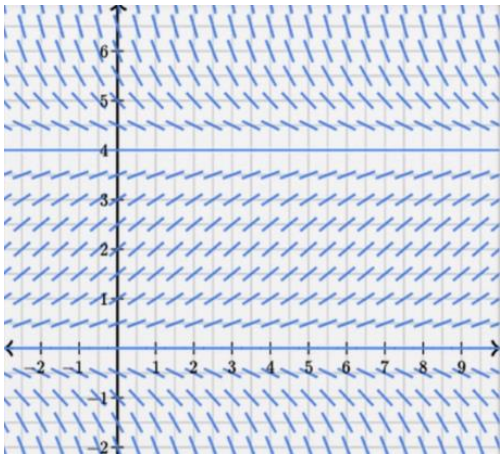
(C) $\frac{dy}{dx} = -x + y$

(D) $\frac{dy}{dx} = x^2 - y$

Example 3:

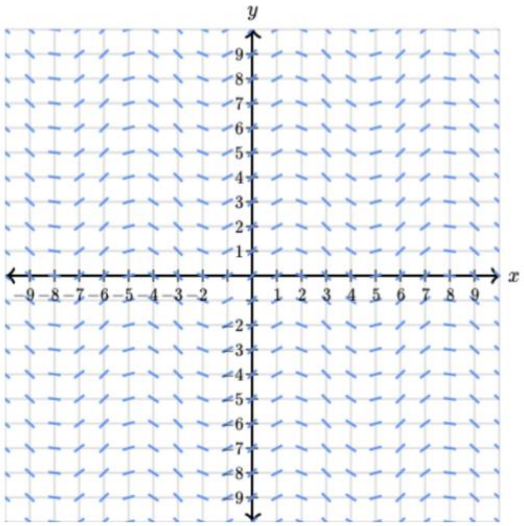
The graph to the right shows a slope field for the function $y = f(x)$.

If the initial condition is (0,6), what is the range of the solution curve $y = f(x)$ for $x \geq 0$?



Example 4:

What is the general solution to the differential equation that generated the slope field to the right?



(A) $x = \cos y + C$

(B) $y = -\cos x + C$

(C) $y = \cos x + C$

(D) $y = -\sin x + C$

(E) $y = \sin x + C$

Example 5

Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$. Describe all points in the *xy*-plane for which the slopes are positive.