Topic 7.5 Approximating solutions Using Euler's Method

Euler's Method

What happens when you can't solve for the particular solution of a differential equation but you need to approximate a $y-\text{value}$ on the curve? The solution to the differential equation **is** the equation of the curve? We have learned previously that sometimes you can take the anti-derivative and using the initial condition derive such equation. But sometime that is very difficult or impossible. **Euler's Method** is a numerical approach to approximate the particular solution of the differential equation.

When using **Euler's method**, we typically use the same **step size** Δx for all the linear approximations. It is common to use a table to keep track of the estimates in each step. Each value of $\frac{dy}{dx}$ is computed from the *x* and *y* values using the differential equation, and each value of *y* is computed from the *y* and $\frac{dy}{dx}$ values in the previous row.

Ex. 1 Consider the differential equation $\frac{dy}{dx} = 3x - 2y$ and $y(0) = 1.5$. Use Euler's method, with a step size of 1 to approximate $y(2)$.

Ex. 2 Consider the differential equation $\frac{dy}{dx} = 2x - y$ and $g(0) = 1$. Use Euler's method, with a step size of .5 to approximate $g(1.5)$.

Ex. 3 Consider the differential equation $\frac{dy}{dx} = x - y$ and $f(3) = 1$. Use Euler's method, with 2 steps to approximate $f(6)$.

*****For a differential equation, **Euler's Method** leads to an **underestimate** when the curve is concave up and an overestimate when the curve is concave down