Topic 7.6 and 7.7
7.6 Finding General Solutions using Separation of Variables
7.7 Finding Particular Solutions Using Initial Value Conditions and Separation of Variables

Separating Variables

Up until now, we have only been able to solve two types of differential equations analytically.

$$y' = f(x) \quad \text{or} \quad \frac{dy}{dx} = f(x) \quad \text{AND} \quad y'' = f(x).$$

In this section we will now solve a more general type of differential equation. The strategy we will use is rewriting the equation so that each variable occurs on only one side of the equation. This strategy is called ***separation of variables***.

The domain of a particular solution to a differential equation is the largest open interval containing the initial value on which the solution is differentiable and satisfies the differential equation.

Example 1: Solve the differential equation $y' = \frac{2x}{y}$.

Original Differential Equation		Rewritten with Variables Separated
$x^2 + 3y\frac{dy}{dx} = 0$	⇒	$3y\,dy = -x^2 dx$
$(\sin x)y' = \cos x$	⇒	$dy = \cot x\,dx$
$\frac{xy'}{e^y + 1} = 2$	⇒	$\frac{1}{e^y + 1}dy = \frac{2}{x}dx$

Example 2: Solve the differential equation: $\frac{dy}{dx} = \frac{4x^3}{\cos y}$.

Finding Particular Solutions

Example 3: Given the differential equation $\frac{dy}{dx} = \sqrt[3]{x}$, find the particular solution determined by the initial condition $y(1) = 2$. (This is an implicit equation so we CAN use the initial-value process I tend to use with the initial condition)

If the derivative is in terms of x and y, you cannot use the method above. On a Free Response question, the following steps are necessary and usually worth 4 or 5 points out of 9.

1. Separates the Variables
2. Takes the Antiderivatives of both sides correctly
3. Uses initial condition to find the constant of integration
4. Solves the equation for y
5. Use equation to evaluate the value (if applicable)

Example 4: Given the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$, find the particular solution determined by the initial condition $y(0) = -2$. (This is an explicit equation so we CANNOT use the initial-value process I tend to use. Must find C)

Example 5: Given the differential equation $(x^2 + 4)\frac{dy}{dx} = xy$, find the particular solution determined by the initial condition $y = 2$ when $x = -3$. (This is an explicit equation so we CANNOT use the initial-value process I tend to use. Must find C)

<u>Example 6</u>: Find the equation of the curve that passes through the point (1,3) and has a slope of $\frac{y}{x^2}$ at any point (x, y). Be sure to state the domain of your solution equation. (This is an explicit equation so we CANNOT use the initial-value process I tend to use. Must find C)