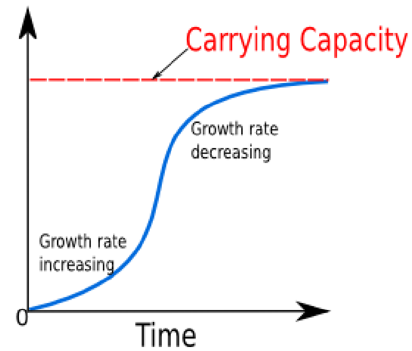
Topic 7.9 Logistic Models with Differential Equations

The differential equation $\frac{dP}{dt} = kP(L-P)$ is a **Logistic Differential Equation**. It describes the growth rate of a population P with carrying capacity L. "The rate of change of a Population is jointly proportional to the size of the Population and the difference between the Population and the carrying capacity"

The **solution** to this differential equation is $P(t) = \frac{L}{1+Ae^{-kLt}}$ AND $\lim_{t \to +\infty} P(t) = L$.

$\frac{d^2P}{dt^2} > 0$; rate of change of Population is increasing
$\frac{d^2P}{dt^2} < 0$; rate of change of Population decreasing
$\frac{d^2P}{dt^2} = 0$; rate of change of Population is increasing the fastest



What you need to be able to do:

1. Make it look like $\frac{dP}{dt} = kP(L-P)$.
2. Find the limit of $P(t)$ as t approaches infinity
3. Tell when the growth is the fastest
4. Given $A, P(t)$, and $\frac{dP}{dt}$, find k.

Example 1: Which ones of the following differential equations model logistic growth? If the equation models logistic growth, identify the carrying capacity of the model.

a.) $\frac{100}{P}\frac{dP}{dt} = \left(1 - \frac{P}{350}\right)$

b.) $\frac{dP}{dt} = \frac{P}{45}\left(3 - \frac{P}{1000}\right)$

c.) $\frac{dy}{dt} = \frac{t}{20}\left(1 - \frac{1}{5000}t\right)$

Example 2: A population of bacteria grows according to the differential equation $\frac{dP}{dt} = 0.05P(1 - .001P)$ When t = 0 days, the population is 300 g.

a.) What is the greatest rate of change, in bacteria per day, of the amount of bacteria?

b.) What are of the values of P for which the population is increasing at a decreasing rate?

Example 3: $\frac{dP}{dt} = \frac{P}{3}\left(1 - \frac{P}{12}\right)$. If $P(0) = 5$, what is $\lim_{t\to\infty} P(t)$?

Example 4: Write the initial value problem that has the solution $P(t) = \frac{4000}{1+7e^{-4t}}$.