Notes 8.1

Example 3: Got Milk?

The cows on a small dairy farm in northern Indiana are milked in the morning during a two-hour period. Milk from the cows are pumped into a storage tank and the volume of milk in the tank at time t is modeled by a strictly increasing, twice-differentiable function, M, where $M(t)$ is measured in gallons and t is measured in minutes. At time $t = 0$ there are 300 gallons of milk in the storage tank. Values of $M(t)$ at selected times t are given in the table.

t (minutes)	0	20	50	90	120
$M(t)$ (gallons)	300	780	1640	3360	4250

a. Estimate $M'(70)$. Indicate the units of measure and explain the meaning of your answer in the context of this problem.

$$m'(70) = \frac{m(90)-m(50)}{3360-1640} = \frac{3360-1640}{40} = \frac{1720}{40} = 43 \text{ gal/min}$$

the amount of milk in the storage tank is increasing by 43 gallons per minute at $t = 70$ minutes

b. Use the data in the table to evaluate $\int_0^{120} M'(t)dt$. Using correct units, interpret the meaning of $\int_0^{120} M'(t)dt$ in the context of the problem.

$$\int_0^{120} m'(t)dt = M(120) - m(0) = 4250 - 300 = 3950 \text{ gallons}$$

3950 gallons of milk have been added to the storage tank from $t = 0$ to $t = 120$ minutes

c. Use a right Riemann sum with four subintervals indicated by the data in the table to approximate $\frac{1}{120}\int_0^{120} M(t)dt$. Explain the meaning of this expression in the context of this problem. Does this approximation overestimate or underestimate the exact value of $\frac{1}{120}\int_0^{120} M(t)dt$? Explain your reasoning.

$$\frac{1}{120}\int_0^{120} m(t)dt \approx \frac{1}{120}\left[4250(120-90) + 3360(90-50) + 1640(50-20) + 780(20-0)\right]$$

$$\approx \frac{1}{120}\left[4250(30) + 3360(40) + 1640(30) + 780(20)\right]$$

$$\approx \frac{1}{120}(326{,}700) = 2722.5 \text{ gallons}$$

The average number of gallons of milk in the storage tank from $t = 0$ minutes to $t = 120$ minutes is approximately 2722.5 gallons. This is an overestimate because $m(t)$ is increasing on $[0, 120]$ and we are using a right Riemann sum approximation.