

Topic 8.3 Using Accumulation Functions and Definite Integrals in Applied Context

AP[®] CALCULUS AB FREE RESPONSE QUESTION

2010 AB/BC 2

(A calculator is allowed)



t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23



Example 1: A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon ($t = 0$) and 8 P.M. ($t = 8$). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \leq t \leq 8$. Value of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

- a. Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

$$E'(6) \approx \frac{E(7) - E(5)}{7 - 5} = \frac{21 - 13}{2} = 4 \text{ hundred entries/hr}$$

- b. Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8} \int_0^8 E(t) dt$. Using correct units, explain the meaning of $\frac{1}{8} \int_0^8 E(t) dt$ in terms of the number of entries.

$$\text{Avg. Value} = \frac{1}{8} \int_0^8 E(t) dt \approx \frac{1}{8} \cdot \frac{1}{a} [2(0+4) + 3(4+13) + 2(13+21) + 1(21+23)]$$

$$\approx \frac{171}{16} \approx 10.688 \text{ hundreds of entries/hr}$$

The average number of entries between Noon and 8 p.m.

- c. At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P , where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \leq t \leq 12$. According to the model, how many entries had not yet been processed by midnight ($t = 12$)?

$$23 - \int_8^{12} P(t) dt = 7 \text{ hundred entries}$$

at $t=8$ ↑

- d. According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

If $P(t)$ is the rate, the candidates are $t = 8 + 12$ AND "maximum RATE"
 when $P'(t) = 0$. $P'(t) = 0$ when $t = 9.1835$ and 10.8165 hours

t	$P(t) = t^3 - 30t^2 + 298t - 976$
8	0
9.1835	5.089
10.8165	rel. min
12	8

The entries are being processed most quickly at $t = 12$ hours at a rate of 8 entries per hour.