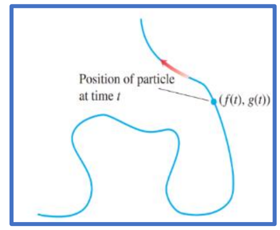
CHA	AP CALCULUS BC	
2	Topic: 9.1	Defining and Differentiating Parametric Equations
Learning Objective CHA-3.G: Calculate derivatives of parametric equations.		

Graphing Parametric Equations



The figure to the right shows the path of a moving particle in the *xy*-plane. Notice that the path fails the vertical line test, so we cannot describe the graph as that of a function.
However, we can often describe the path by a pair of equations, namely $x = f(t)$ and $y = g(t)$, where *f* and *g* are continuous functions.
If this graph truly depicted the path of a moving object, say a bug walking along a glass windowpane, *t* would denote time.

Equations like these can describe more general curves that those described by a single function and they not only provide the graph of the path traced out but also the location of the particle (or bug) $(x, y) = (f(t), g(t))$ at any time *t*.

DEFINITION OF A PLANE CURVE
If *f* and *g* are continuous functions of *t* on an interval *I*, then the equations
$x = f(t)$ and $y = g(t)$
are called parametric equations and *t* is called the parameter. The set of points (x, y) obtained as *t* varies over the interval *I* is called the graph of the parametric equations. Taken together, the parametric equations and the graph are called a plane curve, denoted by *C*.

When sketching a curve by hand represented by parametric equations, you use increasing values of *t*. Thus, the curve will be traced over a specific **direction**. This is called the **orientation** of the curve. You use **arrows** to show the orientation.

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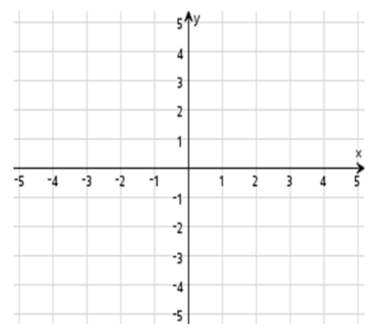
Example 1: Sketching a Parametric Curve - I.
Sketch the curve described by the parametric equations
$x = t^2 - 4$ and $y = \frac{t}{2}, -2 \leq t \leq 3.$

t	-2	-1	0	1	2		3
x							
y							

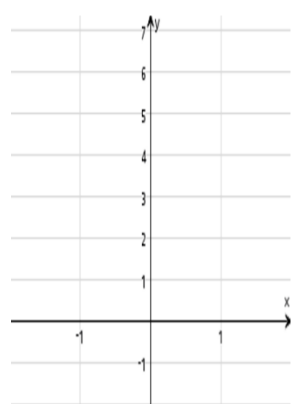


Example 2: Sketching a Parametric Curve – II

Sketch the curve described by the parametric equations

$x = sin\left(\frac{\pi t}{2}\right)$ and $y = t$for $0 \leq t \leq 6.$

t	1	2	3	4	5	6
x						
y						



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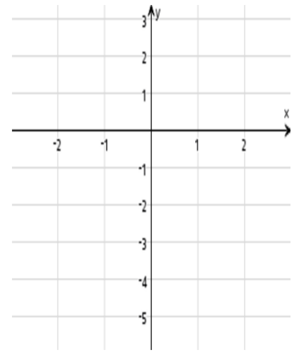
Eliminating Parameters

Many times, when a parametric equation is given, we wish only to sketch the general shape of the plane curve. In that case, we wish to **eliminate the parameter** to create a rectangular equation in the form of $y = f(x)$. The technique to accomplish this is to solve for the parameter in one of the parametric equations (choosing the easiest to do so) and then replacing the result in the other equation.

Example 3: Adjusting the Domain After Eliminating the Parameter

Sketch the curve described by the parametric equations $x = \frac{1}{\sqrt{t+1}}$ and $y = \frac{t}{t+1}, t > -1$ by eliminating the parameter and adjusting the domain of the resulting rectangular equation.



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DEFINITION: SMOOTH CURVE

Let C denote a plane curve represented by the parametric equations $x = f(t)$ and $y = g(t)$ for $a \leq t \leq b$. Suppose each function $x(t)$ and $y(t)$ is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a, b). If both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are both continuous and are never simultaneously 0 on (a, b), then C is called a **smooth curve** on $[a,b]$.

Differentiating Parametric Equations

THEOREM 9.1 PARAMETRIC FORM OF THE DERIVATIVE

If a smooth curve C is given by the equations $x = f(t)$ and $y = g(t)$, then the slope of C at (x, y) is

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \frac{dx}{dt} \neq 0.$$

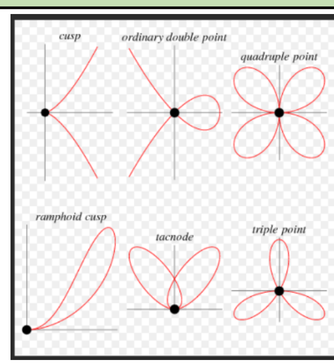
HORIZONTAL & VERTICAL TANGENT LINES

At a number t where $\frac{dy}{dt} = 0$, $\left(\text{but } \frac{dx}{dt} \neq 0\right)$, a smooth curve c has a **horizontal tangent line**.

At a number t where $\frac{dx}{dt} = 0$, $\left(\text{but } \frac{dy}{dt} \neq 0\right)$, a smooth curve c has a **vertical tangent line**.

Singular Points

At a number t where $\frac{dy}{dt} = 0, \frac{dx}{dt} = 0$ simultaneously. See graphs to the right



Example 4: Tangent Line to a Plane Curve

Consider the plane curve defined by the parametric equations $x(t) = 3t^2, y(t) = 2t$.

a.) Find an equation of the tangent line to the curve when $t = 1$.

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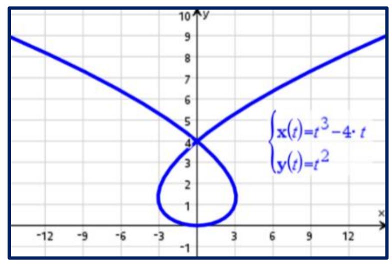
b.) Find all the points on the curve at which the tangent line is vertical.

Example 5: A Curve That Crosses Itself

The plane curve represented by the parametric equations $x(t) = t^3 - 4t, y(t) = t^2$, crosses itself at the point (0, 4), as shown in the graph to the right, and thus has two tangent lines. Find equations for these two lines.



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