CHA	AP CALCULUS BC	
1	Topic: 9.3	Finding Arc Lengths of Curves Given by Parametric Equations

Learning Objective CHA-6.B: Determine the length of a curve in the plane defined by parametric functions, using a definite integral.

Now that we have seen how parametric equations can be used to describe the path of a particle moving in the plane, our next step is to develop a formula for determining the *distance* traveled by the particle along its path.

Recall our arc length formula for a curve defined by *y(x)* on the interval $[x_0, x_1]$.

$$L = \int_{x_0}^{x_1} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

If our curve is represented by the parametric equations, $x = f(t)$ and $y = g(t)$, $a \le t \le b$, and $dx/dt = f'(t) > 0$, you can write

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = \int_a^b \sqrt{1+\left(\frac{dy/dt}{dx/dt}\right)^2}\, dx = \int_a^b \sqrt{\frac{(dx/dt)^2+(dy/dt)^2}{(dx/dt)^2}} \cdot \frac{dx}{dt}\, dt$$

$$= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\, dt = \int_a^b \sqrt{[f'(t)]^2+[g'(t)]^2}\, dt$$

THEOREM 9.3-A ARC LENGTH IN PARAMETRIC FORM

If a smooth curve C is given by the equations $x = f(t)$ and $y = g(t)$, such that C does not intersect itself on the interval $a \le t \le b$ (except possible at the endpoints), then the arc length of C over the interval is given by

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\, dt = \int_a^b \sqrt{[f'(t)]^2+[g'(t)]^2}\, dt$$

NOTE: When applying the arc length formula to a curve, be sure the curve is traced out only once on the interval of integration. For instance, $x = \cos t$ and $y = \sin t$ is traced out once on the interval $0 \le t \le 2\pi$, but is traced out twice on the interval $0 \le t \le 4\pi$.

ARC LENGTH & DISTANCE TRAVELED

The distance a particle travels along a smooth curve on the interval $a \le t \le b$ is synonymous with the arc length of the curve over that same interval. Or,

$$\text{Distance} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\, dt = \int_a^b \sqrt{[f'(t)]^2+[g'(t)]^2}\, dt$$

Example 1: Finding Arc Length for a Parametrically Defined Curve

Find the arc length of the curve defined by $x(t) = t^3 + 2, y(t) = 2t^{9/2}$ on the interval $1 \leq t \leq 3$.

SCAN ME

Scan the QR Code above to watch a video covering Example 1

Example 2: Finding Distance Traveled

A particle moving along a curve in the *xy*-plane is at position $(x(t), y(t))$at time *t*, for $t \geq 0$, where $\frac{dx}{dt} = tan^{-1}(e^{-t}), \frac{dy}{dt} = \frac{3t}{1+4t^3}$. Find the distance traveled by the particle on the time interval $1 \leq t \leq 2$.

SCAN ME

Scan the QR Code above to watch a video covering Example 2