FUN	AP CALCULUS BC	
1	Topic: 9.6	Solving Motion Problems Using Parametric and Vector-Valued Functions
Learning Objective FUN-8.B: Determine values for positions and rates of change in problems involving planar motion.		

We spend a great deal of time in Calculus AB discussing straight line or rectilinear motion ("an object travels along the x-axis and has a position of $s(t) = t^2 - 4t + 5$, etc.."). We can now discuss **motion along curved paths**.

As an object moves along a curve in the plane, the coordinates *x* and *y* of its center of mass are each functions of time *t*. Rather than use *f* and *g* to represent two functions, we write $x = x(t)$ and $y = y(t)$. So, our position vector $r(t)$ is in the form

$$r(t) = x(t)\boldsymbol{i} + y(t)\boldsymbol{j}.$$

The nice thing about doing this is that it is very easy to use derivatives of the vector-valued function **r** to find the object's velocity and acceleration. Remember that velocity and acceleration are both vector quantities having magnitude and direction.

DEFINITION: Velocity and Acceleration

If $x(t)$ and $y(t)$ are twice differentiable functions of *t* and **r** is a vector-valued function given by $r(t) = x(t)\mathrm{i} + y(t)\mathrm{j}$, the velocity vector and acceleration vector at time *t* are

Velocity: $v(t) = r'(t) = x'(t)\boldsymbol{i} + y'(t)\boldsymbol{j}$

Acceleration: $a(t) = r''(t) = x''(t)\boldsymbol{i} + y''(t)\boldsymbol{j}$

Speed: $\|\boldsymbol{v(t)}\| = \|\boldsymbol{r'(t)}\| = \sqrt{[\boldsymbol{x'(t)}]^2 + [\boldsymbol{y'(t)}]^2}$

Example 1 a- b: Velocity, Acceleration and Speed

A particle moves along a plane curve described by $r(t) = 3\,sin\left(\frac{t}{2}\right)\boldsymbol{i} + 3\,cos\left(\frac{t}{2}\right)\boldsymbol{j}$.

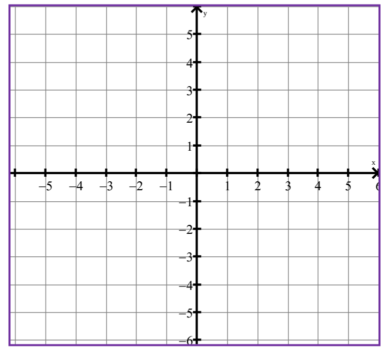
a.) Generate a rectangular equation that describes the particle's motion.

Scan the QR Code above to watch a video covering Example 1 (a and b)

b.) Find the velocity vector, acceleration vector and speed. Interpret the speed.

Example 1 c - d

c.) Show the particle's path on the graph to the right. Be sure to show arrows to indicate direction.

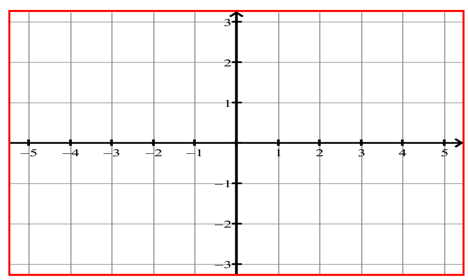


Scan the QR Code above to watch a video covering Example 1 (c and d)

d.) Find the velocity and acceleration at $t = \pi$. Draw the velocity and acceleration vectors on the graph.

Example 2: Motion Along A Curve - I

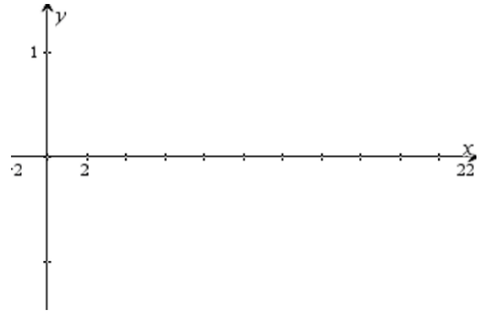
Sketch the path of an object moving along the plane curve given by $\mathrm{r}(t) = (t^2 - 3)\boldsymbol{i} + t\boldsymbol{j}$ and find the velocity and acceleration vectors and speed at $t = 0$ and $t = 2$. Sketch the velocity and acceleration vectors on your curve below.



Scan the QR Code above to watch a video covering Example 2

Example 3: **Motion Along A Curve - II**

a.) Sketch the path of an object moving along the plane curve given by $\mathbf{r}(t) = e^t\mathbf{i} + \ln t\mathbf{j}$.



b.) Find the velocity and acceleration vectors at $t = 1$.

Scan the QR Code above to watch a video covering Example 3

c.) Sketch the velocity and acceleration vectors on your curve above and to the right.

d.) Find the speed at $t = 1$.

e.) What happens to the position, velocity, and acceleration as t approaches infinity?

Example 4: **Motion Along A Curve - III**

A particle starts at rest from the point $(-1,2)$ with an acceleration vector of $\mathbf{a}(t) = \mathbf{i} + 2\mathbf{j}$. Find the location of the particle at $t = 2$.

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Example 5: Motion Along A Curve - IV

A particle with an initial velocity $\mathbf{i}+\mathbf{j}$ and initial position (1, 1) has acceleration vector $\mathbf{a}(t)=-\cos t\mathbf{i}+\sin t\mathbf{j}$
Find the location of the particle at $t=\pi$.

Scan the QR Code above to watch a video covering Example 5