# Notes 6.4 APPROXIMATING AREA UNDER A CURVE

**The two big ideas in Calculus are the <u>tangent line problem</u> and the <u>area problem</u>.**

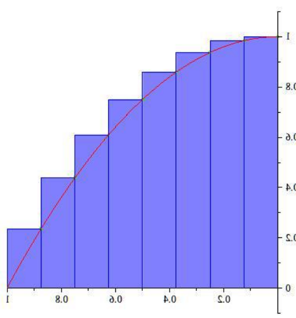
- In the tangent line problem, we saw how the limit process could be applied to the slope of a line to find the slope of a general curve.
- A second classic problem in Calculus is in finding the area of a plane region that is bounded by the graphs of functions. In this case, the limit process is applied to the area of a rectangle to find the area of a general region.

**A basic overview of "areas as limits",**

- In the "limit of rectangles" approach, we take the area under a curve by approximating a collection of <u>inscribed</u> rectangles, <u>circumscribed</u> rectangles, or a more accurate approach of using midpoints. We are interested in finding the area of a region bounded by the $x$-axis which means no portion of its graph on the interval is below the $x$-axis. These methods are commonly known as Riemann Sums. When the number of rectangles is increased without limit we get the actual area. Now known as Integration.

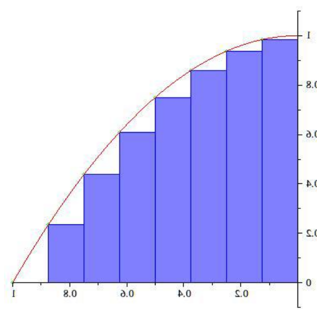
**When the function is increasing:**

Circumscribed Rectangles
(Upper Sum)
Overestimates



**Right end point approximation**

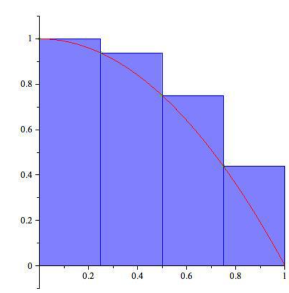
Inscribed Rectangles
(Lower Sum)
Underestimates



**Left end point approximation**

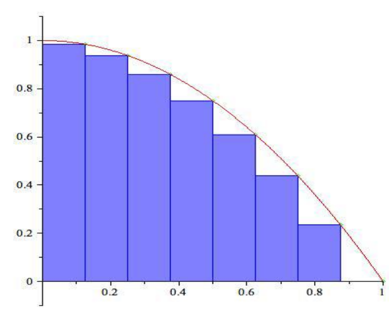
**When the function is decreasing:**

Circumscribed Rectangle
(Upper Sum)
Overestimates

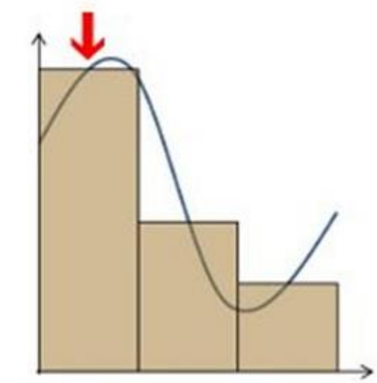


**Left end point approximation**

Inscribed Rectangles
(Lower Sum)
Underestimates



**Right end point approximation**



The middle of the base of the rectangle

**Midpoint Approximation**
Average of the upper and lower sums
More accurate

**<u>Method for approximating area using Riemann Sums:</u>**

**1. Draw a rough sketch of the function over the interval**

**2. Use the formula $\Delta x = \frac{b-a}{n}$ to determine each subinterval, the width of each rectangle**

**3. Compute the areas of each rectangle using the formula $\Delta x \cdot f(x^*)$**

**4. Find the summation of the approximated areas $A = \sum_{k=1}^{n} \Delta x \cdot f(x^*)$**

**Ex.1 Approximate the area under the curve of $f(x) = 2x - 3$ in the interval $[2, 6]$.**

**a.) Use Geometry**

**b) Divide the interval into 4 subintervals of equal length and compute the lower sum (inscribed rectangles)**

**c) Divide the interval into 4 subintervals of equal length and compute the upper sum (circumscribed rectangles)**

**d.) Where parts b-c accurate? Why or Why not?**

**Ex.2 Approximate the area under the curve of $f(x) = -x^2 + 5$ in the interval $[0, 2]$.**

**a.) Divide the interval into 5 subintervals of equal length and compute the lower sum (inscribed rectangles)**

**b) Divide the interval into 5 subintervals of equal length and compute the upper sum (circumscribed rectangles)**

**Ex.2 cont. Approximate the area under the curve of $f(x) = -x^2 + 5$ in the interval $[0, 2]$.**

**c) Use 5 midpoint rectangles**

**d.) Average the areas from a & b. Compare them to c.**

**e.) Use the calculator (Math 9). Where parts a-c accurate? Why or Why not?**