

The following definite integrals are all equal and the variables  $x, t$  and  $y$  are called dummy variables.

$$\int_a^b f(x) dx = \int_a^b f(t) dt = \int_a^b f(y) dy$$

*The value of a definite integral is unaffected if we change the letter used for the variable of integration, but do not change the limits of integration.*

### **The Second Fundamental Theorem of Calculus**

Let  $f$  be a continuous function on an interval  $I$ , and let  $a$  be any point in  $I$ . If  $F$  is defined by  $F(x) = \int_a^x f(t) dt$  then  $F'(x) = f(x)$  at each point  $x$  in the interval  $I$ .

Expressed as a formula: 
$$\frac{d}{dx} \left[ \int_a^x f(t) dt \right] = f(x)$$

*Where the integrand is continuous, the derivative of a definite integral with respect to its upper limit is equal to the integrand evaluated at the upper limit.*

Two other results of the Second Fundamental Theorem of Calculus

$$(1) \quad \frac{d}{dx} \left[ \int_a^{g(x)} f(t) dt \right] = f(g(x)) \cdot g'(x)$$

$$(2) \quad \frac{d}{dx} \left[ \int_{h(x)}^{g(x)} f(t) dt \right] = f(g(x)) \cdot g'(x) - f(h(x)) \cdot h'(x)$$