6.4 Notes Net Signed Area and Limit Process for Integration

If the function f is continuous on $[a,b]$ then the ***net signed area*** A between $y = f(x)$ and the interval $[a,b]$ is defined by:

$$A = \lim_{n\to+\infty} \sum_{k=1}^{n} f(x_k^*)\cdot\Delta x \;=\; \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} f(x_k^*)\cdot\Delta x_k = \int_a^b f(x)dx$$

It represents **Total Area** if $f(x) \geq 0$ on $[a,b]$ and **Net Signed Area,** if $f(x)$ is not above the x-axis over the entire interval. Net Signed Area is the area above the x-axis minus the area below the x-axis. This is also called the ***definite integral***.

History:

The expression "$f(x)dx$" was interpreted to be the "infinitesimal area" of a rectangle with height $f(x)$ and "infinitesimal" width dx. By "summing" these infinite areas, the entire area under the curve was obtained. The integral symbol "$\int$ " is an "elongated s" that was used to indicate this summation. The definite integral is sometimes referred to as the Riemann integral in honor of German mathematician Bernhard Riemann who formulated many of the basic concepts of integral calculus (because that's what we do in our spare time ☺)

Limit Process for Integration

$\lim_{n\to+\infty} \sum_{k=1}^{n} f(x_k^*)\cdot\Delta x$, **given** x_k^*.

Left endpoints: $x_k^* = a + (k-1)\Delta x$

Right endpoints: $x_k^* = a + k\Delta x$

Midpoints: $x_k^* = a + (k - \frac{1}{2})\Delta x$

Ex. Find the area under the curve using the limit of a Riemann sum and x_k^* as a left endpoint.

$\boldsymbol{f(x) = x^2;\ [0,1]}$

STEP 1: Find Δx in terms of n. (Width of each rectangle)

STEP 2: Find x_k^* in terms of n using the **given** formulas

STEP 3: Find $f(x_k^*)$ in terms of *n*. (Height of each rectangle)

STEP 4: Find the limit of the Area of *n* rectangles by substituting in STEP 1 and 3.

$A = \lim_{n\to+\infty} \sum_{k=1}^{n} f(x_k^*) \cdot \Delta x$

Memorizing the following may save you some time

THEOREM:

1.) $\lim_{n\to+\infty} \frac{1}{n} \sum_{k=1}^{n} 1 = 1$

2.) $\lim_{n\to+\infty} \frac{1}{n^2} \sum_{k=1}^{n} k = \frac{1}{2}$

3.) $\lim_{n\to+\infty} \frac{1}{n^3} \sum_{k=1}^{n} k^2 = \frac{1}{3}$

4.) $\lim_{n\to+\infty} \frac{1}{n^4} \sum_{k=1}^{n} k^3 = \frac{1}{4}$