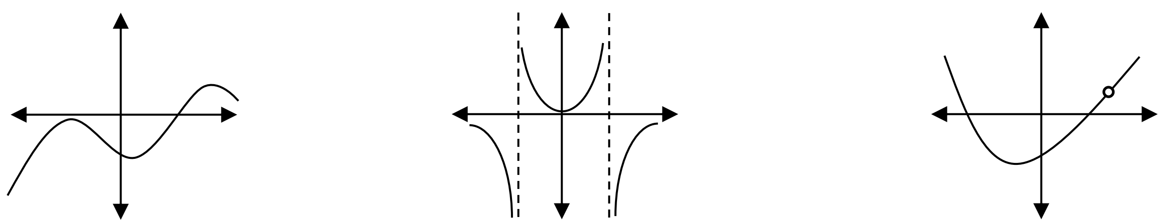
Algebra 2 **Rational Functions and their Graphs** **Name:**

Rational Function

$$f(x) = \frac{P(x)}{Q(x)}$$

where $P(x)$ and $Q(x)$ are polynomial functions and $Q(x) \neq 0$

Continuous graph – a graph that can be drawn with a pencil that never leaves the paper.



continuous not continuous not continuous

Finding points of Discontinuity

To find points of discontinuity – find the values that will make the denominator = 0.

Example: $y = \frac{3}{x^2 - x - 12}$ points of discontinuity: $x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x - 4 = 0$ and $x + 3 = 0$

$x = 4$ and $x = -3$

Example: $y = \frac{2x}{3x^2+4}$ points of discontinuity:

Finding Vertical Aysmptotes and Holes

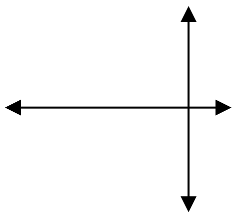
- The rational function $f(x) = \frac{P(x)}{Q(x)}$ has a **point of discontinuity** for each real zero of $Q(x)$
- If $P(x)$ and $Q(x)$ have no common real zeros, then the graph of $f(x)$ has a **vertical asymptote** at each real zero of $Q(x)$
- If $P(x)$ and $Q(x)$ have a common real zero a, then there is a **hole** in the graph at $x = a$ or a **vertical asymptote** at $x = a$ if there is an additional zero in the denominator

Examples: Describe the vertical asymptotes and holes for the graph of each rational function:

a) $y = \frac{x-7}{(x+1)(x+5)}$

discontinuity at $x = -1$ and $x = -5$ there are no common zeros

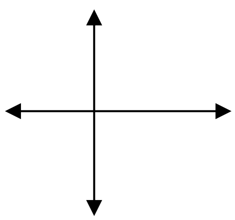
so there are vertical asymptotes at $x = -1$ and $x = -5$



b) $y = \frac{x(x-3)}{(x-3)}$

discontinuity at $x = 3$ $x = 3$ is a common zero

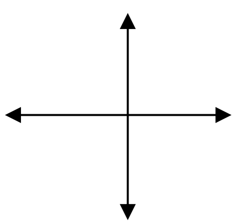
so there is a hole in the graph at $x = 3$



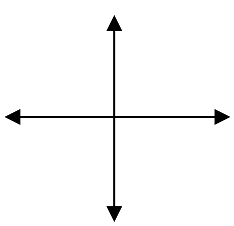
c) $y = \frac{(x-6)(x+9)}{(x+9)(x+9)(x-6)}$

discontinuity at $x = -9$ and $x = 6$ common zeros at $x = -9$ and $x = 6$

So there is a hole at $x = 6$ and a vertical asymptote at $x = -9$



d) $y = \frac{x(x+2)}{(x+2)(x-1)}$



Finding Horizontal Aysmptotes

- The graph of a rational function has at most **one** horizontal asymptote.
- The graph has a horizontal asymptote at $y = 0$ if the degree of the denominator is greater than the degree of the numerator.
- If the degree of the numerator and the degree of the denominator are equal then there is a horizontal asymptote at $y = \frac{a}{b}$, where a is the leading coefficient of the numerator and b is the leading coefficient of the denominator.
- If the degree of the numerator is greater than the degree of the denominator then there is no horizontal asymptote.

Examples: Find the horizontal asymptote of each rational function

$y = \frac{-4x^2+3x}{2x^2+2}$ $\quad \frac{degree = 2}{degree = 2}$ $\quad$ horizontal asymptote: $y = \frac{-4}{2}\ or\ y = -2$

$y = \frac{3x+5}{x^2-9}$ $\quad \frac{degree = 1}{degree = 2}$ $\quad$ horizontal asymptote: $y = 0$

$y = \frac{4x^4+3x^2}{3x^2-3}$ $\quad \frac{degree = 4}{degree = 2}$ $\quad$ no horizontal asymptote

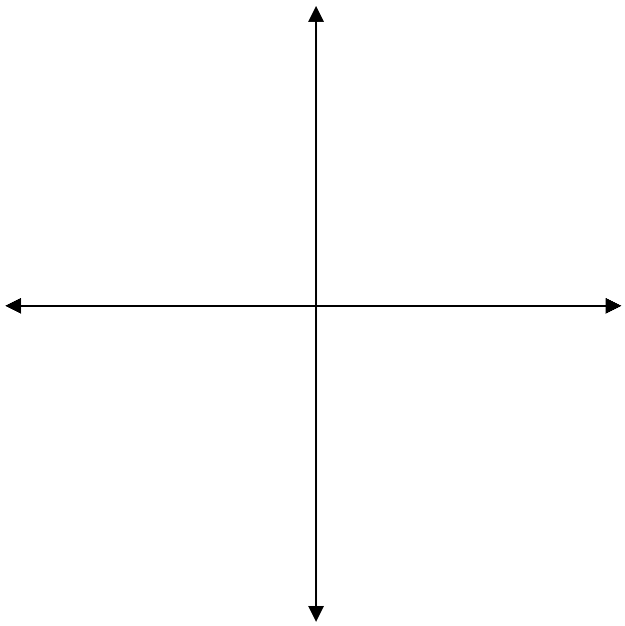
$y = \frac{-15x^3+3x^2-6x}{5x^3-1}$

$y = \frac{7-3x^5}{6x^2+1}$

$y = \frac{x+3}{(x+1)(x-2)}$

Sketching Graphs of Rational Functions

$$y = \frac{x+1}{(x-3)(x+2)}$$



$$y = \frac{2x^2 + x - 1}{x^2 - 2x - 3}$$