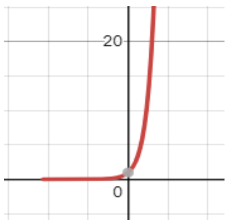
What is an Interval of Convergence?

1. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

Q: Why is the Interval of Convergence $(-\infty, +\infty)$?

- On a graphing calculator we can graph $y = e^x$.
- It would take forever (literally) to input the expanded form of the power series into our calculator.
- It is <u>impossible</u> to graph the closed form to ∞.
- You <u>can</u> graph the closed form using some large number other than ∞.
- Let's try 300, it may take your calculator awhile.

$$y = \sum_{n=0}^{300} \frac{x^n}{n!}$$

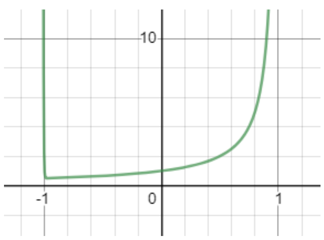


If I were able to go to ∞ or add an infinite number of terms, the graph would be equal to $\boldsymbol{y = e^x}$ for all x. Thus, the **interval of convergence** is $(-\infty, +\infty)$.

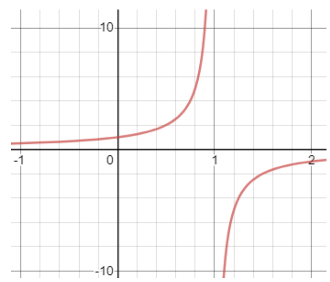
2. $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \ldots = \sum_{n=0}^{\infty} x^n$

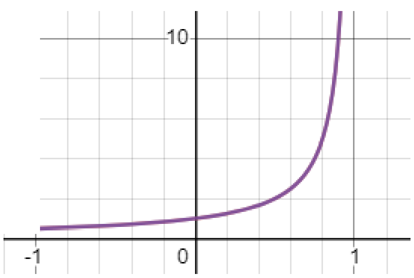
Q: Why is the Interval of Convergence $(-1, 1)$?

$$y = \sum_{n=0}^{300} x^n$$



$$y = \frac{1}{1-x}$$



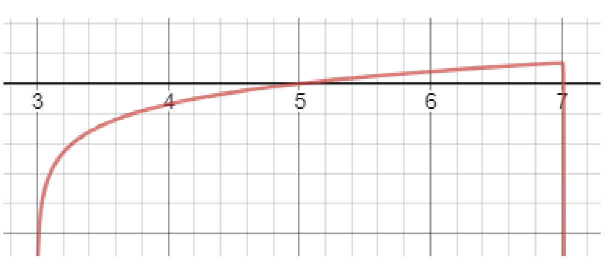


The overlap is the **interval of convergence** $(\mathbf{-1, 1})$. As you can see the endpoints are not included and the center is 0. This graph supports the fact the **radius of convergence** is 1.

3. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n2^n}$

Q: Why is the center 5 and how do I know the Interval of convergence when it's not equal to any function I know?

$$y = \sum_{n=1}^{1000} \frac{(-1)^{(n+1)}(x-5)^n}{n \cdot 2^n}$$



You can see the graph converges between (3,7). Therefore, your **center** must be 5 with a **radius of convergence** of 2. The endpoints would need to be tested. See Power Series 1 Notes, EX 3b.